THE

TRANSACTIONS

OF THE

ROYAL IRISH ACADEMY.

VOLUME XXX.—PART IV.

ON THE RELATIVE POSITIONS OF 223 STARS IN THE CLUSTER χ PERSEI AS DETERMINED PHOTOGRAPHICALLY.

BY SIR ROBERT BALL, LL.D., F.R.S., AND ARTHUR A. RAMBAUT, M.A., D. Sc.

PLATE XVIII.

DUBLIN:

PUBLISHED AT THE ACADEMY HOUSE, 19, DAWSON-STREET.

SOLD ALSO BY

HODGES, FIGGIS, & CO., LTD., GRAFTON-STREET.

AND BY WILLIAMS & NORGATE,

LONDON:
14, Henrietta-street, Covent Garden.

EDINBURGH:
20, South Frederick-street.

1893.

IV.

ON THE RELATIVE POSITIONS OF 223 STARS IN THE CLUSTER χ PERSEI AS DETERMINED PHOTOGRAPHICALLY.

By SIR ROBERT BALL, LL.D., F.R.S., and ARTHUR A. RAMBAUT, M.A., D. Sc. Plate XVIII.

[Read April 25, 1892.]

In the year 1888, a splendid silvered-glass reflecting telescope was presented to the Dunsink Observatory by Dr. Isaac Roberts. This fine instrument, containing a 15-inch With mirror, was employed by Dr. Roberts for some time at the commencement of his now memorable researches in stellar photography, and was afterwards replaced by a larger instrument by Grubb, of two feet aperture, with which most of his celebrated photographs have been obtained.

In mounting this telescope at Dunsink, it was intended to devote it principally to the investigation of stellar parallax by the aid of photography. The prosecution of these researches has, however, been considerably delayed by the difficulties experienced in getting a sufficiently even motion with the driving clock which was supplied by the makers of the mechanical parts of the instrument. These difficulties, and the means taken to obviate them, have been described in the annual reports of the Observatory published in the February number of the *Monthly Notices* of the Royal Astronomical Society for 1891 and 1892.

The mounting of this telescope is of Cook's standard equatorial pattern. Attached to the main tube is the pilot telescope of 5 inches aperture,

the eye-end of which is fitted with an eye-piece movable in two slides at right angles to each other, the arrangement being exactly similar to that applied by Sir Howard Grubb to the standard equatorials used in the photographic survey of the heavens. The declination circle reader, the slow motion in declination, and the electric slow motion in R. A., are all of the construction employed by Sir Howard Grubb in the instruments referred to. The frame which carries the dark slide is suspended by means of three pairs of watch-springs, the planes of which pass through the axis of the tube, so that only their edges obstruct the rays passing down the tube, and the frame is so placed that the sensitive film falls in the principal focal plane of the mirror.

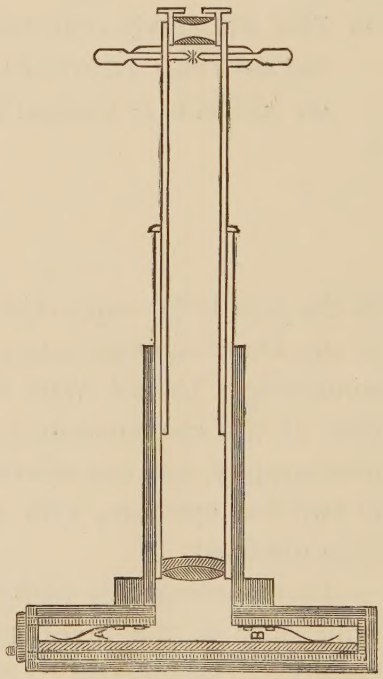

FIG. 1.

The adjustment of the plate and mirror is made very easily by an adaptation of Dr. G. Johnstone Stoney's collimator. This consists of a small telescope of about $\frac{3}{4}$-inch aperture (see adjoining figure) of which the eye-piece is attached to a long draw tube, and in the focus of which are placed two platinum points between which a spark can be made to pass by means of a small electric machine such as is used in electric gas-lighters. We have, then, a piece of patent plate glass, silvered on the back as in the case of an ordinary looking-glass, of the same size as the photographic plates (viz. $3\frac{1}{4} \times 3\frac{1}{4}$) inserted in a special slide, in the back of which is a hole of the same size as the aperture of the collimator. When the slide is inserted in the plate-holder frame, the reflecting surface occupies precisely the position of the sensitive film. At the back of the plate-holder, and at right angles to it, is the focussing tube in which the collimator is inserted, so that when the latter is focussed for parallel rays, and a current produced, the spark and its image are seen in proximity in the field of the collimator,

and by means of the adjusting screws at the back of the plate-holder, they can be made to coincide. In this way the photographic plate can be set accurately at right angles to the axis of the collimator. The slide is then withdrawn, and the eye-piece of the collimator is pulled out until the *surface* of the mirror is in focus as viewed through it. The centre of the mirror is marked by a small cross, and the plate-holder, with collimator attached, is moved bodily by means of the adjusting screws at the ends of the suspension springs, till this cross appears between the platinum points. We then know that the axis of the collimator passes through the centre of the mirror. If, now, the draw-tube of the collimator, carrying the platinum points, be pushed in till the conjugate (virtual) focus is at a distance from the lens equal to the focal length of the mirror, the rays from the spark will fall upon the mirror as if coming from the centre of curvature, and if the mirror is approximately in adjustment, the image of the spark will be found in the field of the collimator, and can be made to coincide with the spark itself by means of the adjusting screws at the back of the mirror box. When this is the case, the principal axis of the mirror passes through the centre of the plate, and is at right angles to it.

For the instrument by which the photographs are measured we are indebted to the Fund for Scientific Research placed at the disposal of the Royal Irish Academy. This instrument was made by Messrs. Troughton and Simms, and is similar to that made by the same firm for Professor Pritchard, at Oxford. It consists of a microscope with a pair of intersecting spider lines at the focus, the stage of which is movable by means of micrometer screws in two directions at right angles to each other. These screws have twenty threads to the inch, and a range of a little more than two inches. The head of each screw is divided into fifty parts, and is geared by means of a pair of toothed wheels to a subsidiary drum which rotates five times as rapidly. These drums are divided into one hundred parts, each of which can be read by estimation to tenths. We have thus one of these ultimate divisions equal to $\frac{1}{100,000}$th of an inch.

The stage is also furnished with a position circle divided from 0° to 360° into spaces of 10′ each, which is read by two opposite verniers, each divided

to 10″, and is provided with a clamp and tangent screw for fine setting. The graduations on the circle run in the same direction as the figures on the face of a watch.

For adjusting any point on the plate at the centre of the position circle the stage is further capable of a rotation with regard to it, and of motions of translation in two mutually perpendicular directions by means of fine opposing screws. The instrument was originally made to carry a circular plate, but has been adapted for a $3\frac{1}{4} \times 3\frac{1}{4}$ square plate, such as our telescope takes, by a slight alteration of the plate-holder.

This instrument measures either the rectangular or polar co-ordinates of the points on the plate, but in the observations given below we have only employed the former system. It is, however, necessary to convert these co-ordinates into differences of R. A. and declination before the measures can be of use. We must now investigate the formulæ by which this conversion can be accomplished.

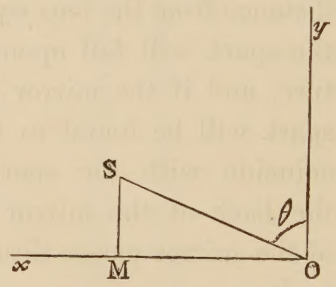

FIG. 2.

We shall suppose that the plate has been adjusted in the manner described above, so that the principal axis of the mirror is normal to it, and that the point in which this axis pierces the plate (which is the *centre of the plate*) has been determined, also that the line on the plate corresponding to the meridian passing through the *centre* is known.

Let *O* (in fig. 2) represent the centre of the plate, *Oy* the projection of the meridian passing through it. Let *S* be the position of a star on the plate, and *Ox* a line at right angles to *Oy*. Join *OS* and drop *SM* perpendicular to *Ox*. Then, *OM* and *SM* are the *x* and *y* of the point *S* as measured in the microscope.

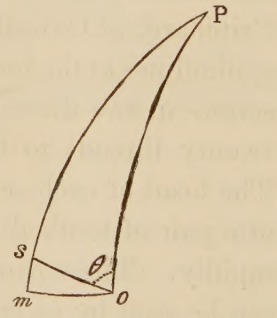

FIG. 3.

Again (in fig. 3), let *o* represent the point of the sky corresponding to the centre of the plate, *P* the pole, and *s* the star.

Let us further denote by

α, δ the R. A. and declination of the centre,

α', δ' the R. A. and declination of the star,

ρ, θ the distance and position angle of s as measured from o, and by

r the distance between the centres of the mirror and plate.

We have, then, the angle yOS = the angle $Pos = \theta$, $os = \rho$ and $OS = r \tan \rho$.

Also $$x = r \tan \rho \sin \theta$$

and $$y = r \tan \rho \cos \theta \qquad (1)$$

from figure 2. But from figure 3 we have

$$\sin \rho \sin \theta = \cos \delta' \sin (\alpha' - \alpha),$$

$$\sin \rho \cos \theta = \cos \delta \sin \delta' - \sin \delta \cos \delta' \cos (\alpha' - \alpha),$$

and $$\cos \rho = \sin \delta \sin \delta' + \cos \delta \cos \delta' \cos (\alpha' - \alpha),$$

whence we obtain

$$x = r \cdot \frac{\cos \delta' \sin (\alpha' - \alpha)}{\sin \delta \sin \delta' + \cos \delta \cos \delta' \cos (\alpha' - \alpha)}$$

and $$y = r \cdot \frac{\cos \delta \sin \delta' - \sin \delta \cos \delta' \cos (\alpha' - \alpha)}{\sin \delta \sin \delta' + \cos \delta \cos \delta' \cos (\alpha' - \alpha)} \qquad (2)$$

From the second of these we find

$$\cos (\alpha' - \alpha) = \tan \delta' \cdot \frac{r \cos \delta - y \sin \delta}{r \sin \delta + y \cos \delta},$$

whence

$$\sin \delta \sin \delta' + \cos \delta \cos \delta' \cos (\alpha' - \alpha)$$

$$= \sin \delta \sin \delta' + \cos \delta \cos \delta' \tan \delta' \cdot \frac{r \cos \delta - y \sin \delta}{r \sin \delta + y \cos \delta}$$

$$= \frac{r \sin \delta'}{r \sin \delta + y \cos \delta}.$$

Substituting this in the first of equations (2), we find, after division by r,

$$(r \sin \delta + y \cos \delta) \cos \delta' \sin (\alpha' - \alpha) = x \sin \delta',$$

or, $$\sin (\alpha' - \alpha) = \tan \delta' \cdot \frac{x}{r \sin \delta + y \cos \delta}.$$

We thus obtain

$$\left.\begin{aligned} \tan(\alpha' - \alpha) &= \frac{x}{r\cos\delta - y\sin\delta}, \\ \text{and} \qquad \tan\delta' &= \frac{r\sin\delta + y\cos\delta}{x}\sin(\alpha' - \alpha), \end{aligned}\right\} \tag{3}$$

from which α' and δ' might be obtained.

Also, from the equations (2) we obtain, by division,

$$\frac{y}{x} = \frac{\cos\delta\sin\delta' - \sin\delta\cos\delta'\cos(\alpha' - \alpha)}{\cos\delta'\sin(\alpha' - \alpha)}. \tag{4}$$

This equation would enable us to orient the plate, being given the R. A. and declination of the centre, and of any star on the plate. For this purpose it would be necessary to proceed as follows:—Determine $\tan\phi = \frac{y}{x}$ from the above. Place the plate so that the line joining the centre with the star in question is parallel to the horizontal screw. Then, by means of the position circle, rotate the plate through an angle equal to ϕ. It will then be set for measurement from the centre.

In general, however, the centre of the plate is not marked with the precision that this method would require, and it will be necessary to proceed differently. The equations (3) also are in a form which would be found very inconvenient in practice. But from equations (2) approximate formulæ may be obtained, by means of which the values of $\alpha' - \alpha$ and $\delta' - \delta$ may be computed with great facility.

We have, from equations (2), denoting $\alpha' - \alpha$ and $\delta' - \delta$ by $\Delta\alpha$, $\Delta\delta$, respectively,

$$x = r\,.\,\frac{\cos\delta'\sin\Delta\alpha}{\cos\rho},$$

$$\text{and} \qquad y = r\,.\,\frac{\cos\delta\sin\delta' - \sin\delta\cos\delta'\cos\Delta\alpha}{\cos\rho},$$

$$\text{and} \qquad x^2 + y^2 = r^2\tan^2\rho.$$

$$\text{Hence,} \qquad y\cos\rho = r\sin\Delta\delta + r\sin\delta\cos\delta'(1 - \cos\Delta\alpha),$$

or, retaining only the third powers of $\Delta\alpha$ and $\Delta\delta$,

$$\sin \Delta\delta = \frac{y}{r} \cos \rho - \sin \delta \cos \delta' . \frac{\Delta\alpha^2}{2}.$$

Also,
$$\Delta\alpha = \sin^{-1}\left(\frac{x \cos \rho}{r \cos \delta'}\right)$$

$$= \frac{x \cos \rho}{r \cos \delta'} + \tfrac{1}{6}\left(\frac{x \cos \rho}{r \cos \delta'}\right);$$

therefore
$$\sin \Delta\delta = \frac{y}{r} \cos \rho - \sin \delta \cos \delta' \frac{x^2 \cos^2 \rho}{2r^2 \cos^2 \delta'}.$$

But,
$$\cos \rho = \frac{1}{\sqrt{1 + \tan^2 \rho}} = \frac{1}{\sqrt{1 + \dfrac{x^2 + y^2}{r^2}}} = 1 - \tfrac{1}{2} . \frac{x^2 + y^2}{r^2};$$

therefore
$$\sin \Delta\delta = \frac{y}{r}\left(1 - \tfrac{1}{2} \frac{x^2 + y^2}{r^2}\right) - \tfrac{1}{2} \frac{\sin \delta}{\cos \delta'} \frac{x^2}{r^2}.$$

But
$$\frac{\sin \delta}{\cos \delta'} = \frac{\sin \delta}{\cos (\delta + \Delta\delta)} = \frac{\sin \delta}{\cos \delta - \Delta\delta \sin \delta}$$

$$= \tan \delta \, (1 + \Delta\delta \tan \delta)$$

$$= \tan \delta + \frac{y}{r} \tan^2 \delta \, ;$$

whence, substituting this value, we find

$$\sin \Delta\delta = \frac{y}{r}\left(1 - \tfrac{1}{2} \frac{x^2 + y^2}{r^2}\right) - \tfrac{1}{2} \frac{x^2}{r^2}\left(\tan \delta + \frac{y}{r} \tan^2 \delta\right),$$

and, finally,

$$\Delta\delta = \frac{y}{r}\left(1 - \tfrac{1}{2} \frac{x^2 + y^2}{r^2}\right) - \tfrac{1}{2} \frac{x^2}{r^2}\left(\tan \delta + \frac{y}{r} \tan^2 \delta\right) + \tfrac{1}{6} \frac{y^3}{r^3}$$

$$= \frac{y}{r} - \tfrac{1}{2} \frac{x^2}{r^2} \tan \delta - \tfrac{1}{2} \frac{x^2 y}{r^3} \sec^2 \delta - \tfrac{1}{3} \frac{y^3}{r^3}. \qquad (5)$$

Again, $$\sin \Delta\alpha \cos \delta' = \frac{x}{r} \cos \rho,$$

but, $$\cos \delta' = \cos (\delta + \Delta\delta) = \cos \delta \left(1 - \frac{\Delta\delta^2}{2}\right) - \Delta\delta \sin \delta$$

$$= \cos \delta \, (1 - \Delta\delta \tan \delta - \tfrac{1}{2} \Delta\delta^2).$$

Therefore,

$$\sin \Delta\alpha \cos \delta = \frac{x}{r} \cos \rho \, [1 + \Delta\delta \tan \delta + \tfrac{1}{2} \Delta\delta^2 + \Delta\delta^2 \tan^2\delta]$$

$$= \frac{x}{r} \left[1 - \tfrac{1}{2} \frac{x^2 + y^2}{r^2}\right] \left[1 + \left(\frac{y}{r} - \tfrac{1}{2} \frac{x^2}{r^2} \tan \delta\right) \tan \delta + \frac{y^2}{r^2} (\tfrac{1}{2} + \tan^2\delta)\right],$$

whence,

$$\sin \Delta\alpha = \frac{x}{r} \sec \delta \left[1 + \frac{y}{r} \tan \delta - \tfrac{1}{2} \frac{x^2}{r^2} \sec^2\delta + \frac{y^2}{r^2} \tan^2\delta\right],$$

and, finally,

$$\Delta\alpha = \frac{x}{r} \sec \delta + \frac{xy}{r^2} \sec \delta \tan \delta - \tfrac{1}{3} \frac{x^3}{r^3} \sec^3\delta + \frac{xy^2}{r^3} \sec \delta \tan^2\delta. \qquad (6)$$

Equations (5) and (6) are thus in a most convenient form for computing the differences of R. A. and declination from the *centre of the plate*, i. e. the point where the principal axis of the mirror intersects it. There is, however, no very obvious way of determining on each plate the exact position of the centre, and it will, therefore, be necessary to investigate how far an erroneous determination of it will affect the resulting differences, $\Delta\alpha$ and $\Delta\delta$; or, in other words, with what degree of accuracy we must determine the centre.

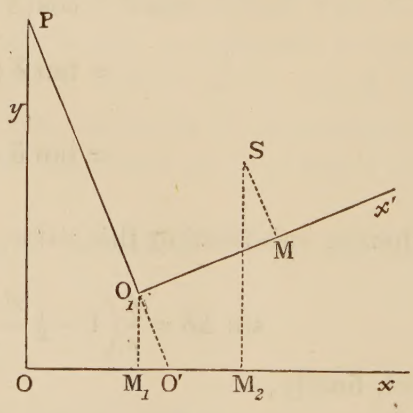

FIG. 4.

Let O (in fig. 4) be the centre of the plate, and OP the projection of the meridian passing through it. Let also P be the projection on the plane of the plate of the north pole of the heavens. Let O_1 be any point on the plate, and O_1P the projection of the meridian passing through it. This will, of

course, be a straight line. Let, now, S be the position of any star on the plate, and drop the perpendiculars O_1M_1, SM_2 on Ox, and SM on O_1x'.

If we denote by x_1, y_1, and x_2, y_2, the co-ordinates of O_1 and S, respectively referred to Ox and OP, by x, y, the co-ordinates of S referred to O_1x' and O_1P, and by δ_1 and δ_2 the declination of O_1 and S respectively, we shall have, from equations (5) and (6), as the true values of $\delta_2 - \delta_1$ and $\alpha_2 - \alpha_1$,

$$\delta_2 - \delta_1 = \frac{y_2 - y_1}{r} - \tfrac{1}{2}\frac{x_2^2 - x_1^2}{r^2}\tan\delta - \tfrac{1}{2}\cdot\frac{x_2^2y_2 - x_1^2y_1}{r^3}\sec^2\delta - \tfrac{1}{3}\frac{(y_2^3 - y_1^3)}{r^3},$$

and

$$\alpha_2 - \alpha_1 = \frac{x_2 - x_1}{r}\sec\delta + \frac{x_2y_2 - x_1y_1}{r^2}\sec\delta\tan\delta - \tfrac{1}{3}\frac{x_2^3 - x_1^3}{r^3}\sec^3\delta + \frac{x_2y_2^2 - x_1y_1^2}{r^3}\sec\delta\tan^2\delta.$$

But if we had erroneously taken O_1 as centre, and applied the formulæ (5) and (6) directly to the measures from this origin, we should have found

$$\delta_2 - \delta_1 = \frac{y}{r} - \tfrac{1}{2}\frac{x^2}{r^2}\tan\delta_1 - \tfrac{1}{2}\frac{x^2y}{r^3}\sec^2\delta_1 - \tfrac{1}{3}\frac{y^3}{r^3}, \qquad (7)$$

and

$$\alpha_2 - \alpha_1 = \frac{x}{r}\sec\delta_1 + \frac{xy}{r^2}\sec\delta_1\tan\delta_1 - \tfrac{1}{3}\frac{x^3}{r^3}\sec^3\delta_1 + \frac{xy^2}{r^3}\sec\delta_1\tan^2\delta_1. \qquad (8)$$

If ψ be the angle between OP and O_1P, we have

$$x = (x_2 - x_1)\cos\psi + (y_2 - y_1)\sin\psi$$

$$y = -(x_2 - x_1)\sin\psi + (y_2 - y_1)\cos\psi.$$

Also, if PO_1 (fig. 4) be produced to O'

$$\tan\psi = \frac{OO'}{OP},$$

and if opo' (fig. 5) be the spherical triangle of which OPO' is the projection, we have

$$\tan oo' = \cos\delta\tan(\alpha_1 - \alpha)$$

and

$$\tan op = \cot\delta.$$

Fig. 5.

But $OO' = r\tan oo'$, and $OP = r\tan op$; therefore $\tan\psi = \sin\delta\tan(\alpha_1 - \alpha)$.

Also,

$$\tan\delta_1 = \tan(\delta + \delta_1 - \delta) = \frac{\tan\delta + (\delta_1 - \delta)}{1 - (\delta_1 - \delta)\tan\delta}$$

$$= \tan\delta + (\delta_1 - \delta)\sec^2\delta + (\delta_1 - \delta)^2 \tan\delta \sec^2\delta$$

$$= \tan\delta + \frac{y_1}{r}\sec^2\delta + \ldots$$

Again,

$$\sec^2\delta_1 = 1 + \tan^2\delta_1 = 1 + \tan^2\delta + 2(\delta_1 - \delta)\sec^2\delta\tan\delta + (\delta_1 - \delta)^2\sec^2\delta(1 + 3\tan^2\delta)$$

$$= \sec^2\delta\left[1 + 2(\delta_1 - \delta)\tan\delta + (\delta_1 - \delta)^2(1 + 3\tan^2\delta)\right];$$

whence

$$\sec\delta_1 = \sec\delta\left[1 + (\delta_1 - \delta)\tan\delta + \tfrac{1}{2}(\delta_1 - \delta)^2(1 + 2\tan^2\delta)\right]$$

$$= \sec\delta\left[1 + \frac{y_1}{r}\tan\delta - \tfrac{1}{2}\frac{x_1^2}{r^2}\tan^2\delta + \tfrac{1}{2}\frac{y_1^2}{r^2}(1 + 2\tan^2\delta)\right].$$

We have, also,

$$\psi = \sin\delta\,(\alpha' - \alpha) = \frac{x_1}{r}\tan\delta + \frac{x_1 y_1}{r^2}\tan^2\delta;$$

whence,

$$\sin\psi = \frac{x_1}{r}\tan\delta + \frac{x_1 y_1}{r^2}\tan^2\delta,$$

and

$$\cos\psi = 1 - \tfrac{1}{2}\frac{x_1^2}{r^2}\tan^2\delta.$$

We thus find for x and y the values

$$x = (x_2 - x_1)\left(1 - \tfrac{1}{2}\frac{x_1^2}{r^2}\tan^2\delta\right) + (y_2 - y_1)\left(\frac{x_1}{r}\tan\delta + \frac{x_1 y_1}{r^2}\tan^2\delta\right)$$

$$y = -(x_2 - x_1)\left(\frac{x_1}{r}\tan\delta + \frac{x_1 y_1}{r^2}\tan^2\delta\right) + (y_2 - y_1)\left(1 - \tfrac{1}{2}\frac{x_1^2}{r^2}\tan^2\delta\right).$$

Whence, substituting these values of x and y, with the values found above for $\tan\delta_1$ and $\sec\delta_1$, in equation (7), we find

$$\delta_2 - \delta_1 = -\frac{(x_2 - x_1)}{r}\left(\frac{x_1}{r}\tan\delta + \frac{x_1 y_1}{r^2}\tan^2\delta\right) + \frac{(y_2 - y_1)}{r}\left(1 - \tfrac{1}{2}\frac{x_1^2}{r^2}\tan^2\delta\right)$$

$$- \tfrac{1}{2}\left[\frac{(x_2 - x_1)}{r} + \frac{(y_2 - y_1)}{r}\frac{x_1}{r}\tan\delta\right]^2\left[\tan\delta + \frac{y_1}{r}\sec^2\delta\right]$$

$$-\tfrac{1}{2}\frac{(x_2-x_1)^2}{r^2}\cdot\frac{(y_2-y_1)}{r}\sec^2\delta-\tfrac{1}{3}\frac{(y_2-y_1)^3}{r^3},$$

which becomes, after some reduction,

$$\delta_2-\delta_1=\frac{y_2-y_1}{r}-\tfrac{1}{2}\cdot\frac{x_2^2-x_1^2}{r^2}\tan\delta-\tfrac{1}{2}\frac{(x_2^2y_2-x_1^2y_1)}{r^3}\sec^2\delta-\tfrac{1}{3}\frac{(y_2^3-y_1^3)}{r^3}$$

$$-\tfrac{1}{2}\left[\frac{x_1^2(y_2+y_1)}{r^2}\frac{}{r}-\frac{2x_1x_2y_2}{r^3}-\frac{2y_1y_2}{r^2}\frac{(y_2-y_1)}{r}\right].$$

The last term in this expression is, therefore, the error affecting $\delta_2-\delta_1$, which is caused by an erroneous assumption of the centre. But this expression, even in the extreme case where $x_1=y_1=-300''$ and $x_2=y_2=1800''$ will never exceed $0''{\cdot}05$. The error produced in the value of $(\alpha_2-\alpha_1)\cos\delta$ by an erroneous assumption of the centre will also be a small quantity of the same order of magnitude. We thus see that so long as the assumed centre is within five or six minutes of arc of the real centre of the plate no correction for erroneous centre need be applied. This corresponds to a distance of about one-fifth of an inch on the plates, which leaves a considerable area within which it is easy, without troublesome refinements, to set a star whose image may be taken as the centre of the plate.

It should, however, be noted, that whatever point be selected as centre within the limits permitted to us, it will be necessary to determine the position of the parallel for that point, and to set this line parallel to one of the screws. This appears from the fact, that in what precedes, the axis O_1x' is supposed to be in the direction of the tangent to the parallel at O_1. This is a most important point, a neglect of which would introduce an error containing terms of the second degree, viz. $-x_1(y_1-y_2)\tan\delta\sec\delta$ and $x_1(x_1-x_2)\tan\delta$ in R. A. and declination respectively, the first of which, in the case we have supposed, would amount to $13''{\cdot}3$.

In practice, the method we adopt for determining the parallel is as follows:—Having found the readings of the scales at the eye-end of the pilot telescope corresponding to the centre of the plate, we set the telescope in declination so that the image of some star in the field will pass approximately through this centre, and in R. A., so that the image of the star is situated at about seven and a-half minutes of arc to the following side of

the centre, and expose the plate. We then stop the clock for such time as the star takes to move through fifteen minutes of arc and make another similar exposure. We have thus two images of the star symmetrically placed with regard to the centre, the line joining which is the parallel—not for the star—but for the point which (in either exposure) lies at the middle of the line joining the two star-images. It is, however, convenient to take as origin the centre, not of the chord, but of the arc of the parallel joining the two images, as we shall then have the declination of the adopted centre equal to that of the star in question.

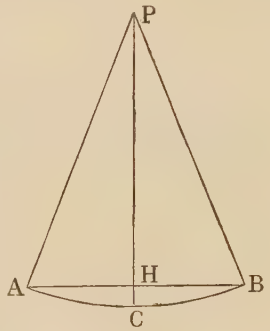

FIG. 6.

If A and B (fig. 6) be the two images of the star, and H the middle point of the line joining them, then we may assume that between A and B the projection of the small circle is itself a circle. We thus find $HC = AH \tan \frac{1}{2}APC$. But if Δt is the time during which the clock is stopped, $\tan APC = \sin \delta \tan \frac{1}{2}\Delta t$, or we may put $APC = \frac{1}{2}\Delta t \sin \delta$, but

$$\Delta t = \frac{AB}{r} \sec \delta, \quad \therefore \frac{1}{2}APC = \frac{1}{4}\frac{AB}{r} \tan \delta,$$

and therefore

$$HC = \frac{1}{8}\frac{AB^2}{r} \tan \delta.$$

To adjust the plate for measurement in the microscope, and to determine the screw-readings of the origin, we proceed then as follows:—We rotate the plate until the intersection of the lines is carried by a movement of the H (the horizontal) screw alone from A to B. This was found rather a tedious process at first when the two images of the star were not in the field of the microscope at the same time, requiring a great deal of screwing backwards and forwards before a perfectly satisfactory adjustment could be obtained.

The following method, however, was found very precise and expeditious. When the plate had been placed approximately correctly, we took a number of readings on both screws of the positions of both images. If the means of these for the image A be H and V, respectively, and for the

image B, H' and V', respectively, it is clear that the angle (Δp) between the line AB and the direction of the H screw is

$$\tan^{-1}\frac{V'-V}{H'-H}=\Delta p.$$

This is, therefore, the correction to the reading of the position circle. When this has been applied it only remains to take the readings H_A, V_A and H_B, V_B for the two images, and we have, if H_0 and V_0 denote the readings corresponding to the centre,

$$H_0=\tfrac{1}{2}(H_A+H_B),$$

and

$$V_0=\tfrac{1}{2}(V_A+V_B)\pm\tfrac{1}{8}\frac{AB^2}{r}\tan\delta.$$

The sign of the correction $\frac{1}{8}\frac{AB^2}{r}\tan\delta$ will depend on the way the plate is placed in the microscope. If it is placed so that the stars appear in the same relative position when viewed through the microscope as they do in an astronomical telescope, and if, as in the case before us, the readings of the V screw increase from the lower to the upper end of the scale the sign of this correction will always be positive. We thus obtain the readings H_0 and V_0 corresponding to the centre of the plate, and we have the declination of the centre (δ) equal to that of the star by which the adjustments have been made. If, then, H and V are the readings of any other star, we have

$$x=H-H_0 \quad \text{and} \quad y=V-V_0,$$

from which, by means of the formulæ (5) and (6), the apparent differences of R.A. and declination from the centre can be obtained, and, consequently, the differences of all the stars on the plate, measured from any one of them, taken as a fundamental star.

We have next to investigate the formulæ by which the relative apparent positions of the stars can be corrected for the effects of refraction, aberration, nutation, and precession. The correction for the last three of these is contained in the usual formulæ,

$$\text{correction to } \Delta\alpha=-[g\cos(G+\alpha)\tan\delta+h\cos(H+\alpha)\sec\delta]\sin 1''\,\Delta\alpha$$
$$-[g\sin(G+\alpha)\sec^2\delta+h\sin(H+\alpha)\sec\delta\tan\delta]\sin 1''\,\Delta\delta,$$

and correction to $\Delta\delta = [g \sin(G+\alpha) + h \sin(H+\alpha) \sin\delta] \sin 1'' \Delta\alpha$
$+ [i \sin\delta - h \cos(H+\alpha) \cos\delta] \sin 1'' \Delta\delta,$

in which $\Delta\alpha$ and $\Delta\delta$ are expressed in seconds of arc, while f, g, h, i, G and H are taken from the Nautical Almanac.

The expression for the refraction will, however, be different from that used to correct measures made with a micrometer of the ordinary form in which the difference of R.A. is found from transits of the stars across a wire placed perpendicular to the parallel, while the difference of declination is measured directly. In that case the stars are observed on the same hour circle, and it is not necessary to take into account the difference of Right Ascension in computing the refraction.

The ordinary expression for the refraction is

$$\beta \tan\zeta$$

where ζ denotes the true zenith distance of the star, and β is a function of ζ. Since, however, the plate does not represent the appearance of the sky at any given moment, but its average appearance for the whole time of exposure, during which the stars will in general have moved through a considerable angle, it will not be necessary to take into account the variation of β from point to point of the plate. It will be sufficient to assume as the law of refraction

$$k \tan\zeta$$

where k is the value of β corresponding to the central star at the middle of the exposure.*

If α, δ be the apparent, and $\alpha+\chi$, $\delta+\psi$ the true R. A. and declination of the central star, and if α', δ' are the apparent R. A. and declination of any other star on the plate, then the true co-ordinates of this star are

$$\alpha' + \chi + \frac{d\chi}{d\alpha}\Delta\alpha + \frac{d\chi}{d\delta}\Delta\delta,$$

and

$$\delta' + \psi + \frac{d\psi}{d\alpha}\Delta\alpha + \frac{d\psi}{d\delta}\Delta\delta.$$

* "The Corrections for Refraction to Measures of Stellar Photographs." By Arthur A. Rambaut, Astronomische Nachrichten, No. 3125.

Hence the corrections to be applied to $\alpha' - \alpha$ and $\delta' - \delta$ are, respectively,

$$\frac{d\chi}{d\alpha}\Delta\alpha + \frac{d\chi}{d\delta}\Delta\delta,$$

and

$$\frac{d\psi}{d\alpha}\Delta\alpha + \frac{d\psi}{d\delta}\Delta\delta.$$

But

$$\chi = -k \tan\zeta \sin\eta \sec\delta,$$

and

$$\psi = -k \tan\zeta \cos\eta,$$

in which η denotes the parallactic angle.

We have also, if ϕ is the latitude of the Observatory, and θ the sidereal time at the middle of the exposure, the following equations:—

$$\sin\zeta \sin\eta = \cos\phi \sin(\theta - \alpha)$$

$$\sin\zeta \cos\eta = \cos\delta \sin\phi - \sin\delta \cos\phi \cos(\theta - \alpha)$$

$$\cos\zeta = \sin\delta \sin\phi + \cos\delta \cos\phi \cos(\theta - \alpha).$$

We find, accordingly,

$$\chi = -k \cdot \frac{\cos\phi \sin(\theta - \alpha)}{\cos\delta \{\sin\delta \sin\phi + \cos\delta \cos\phi \cos(\theta - \alpha)\}},$$

and

$$\psi = -k \cdot \frac{\cos\delta \sin\phi - \sin\delta \cos\phi \cos(\theta - \alpha)}{\sin\delta \sin\phi + \cos\delta \cos\phi \cos(\theta - \alpha)}.$$

Hence,

$$\frac{d\chi}{d\alpha} = k \cdot \frac{\cos\phi}{\cos\delta} \cdot \frac{\cos\delta \cos\phi + \sin\delta \sin\phi \cos(\theta - \alpha)}{\{\sin\delta \sin\phi + \cos\delta \cos\phi \cos(\theta - \alpha)\}^2},$$

$$\frac{d\chi}{d\delta} = k \cdot \frac{\cos\phi \sin(\theta - \alpha)}{\cos^2\delta} \cdot \frac{\cos 2\delta \sin\phi - \sin 2\delta \cos\phi \cos(\theta - \alpha)}{\{\sin\delta \sin\phi + \cos\delta \cos\phi \cos(\theta - \alpha)\}^2},$$

$$\frac{d\psi}{d\alpha} = k \cdot \frac{\sin\phi \cos\phi \sin(\theta - \alpha)}{\{\sin\delta \sin\phi + \cos\delta \cos\phi \cos(\theta - \alpha)\}^2},$$

and

$$\frac{d\psi}{d\delta} = k \cdot \frac{\sin^2\phi + \cos^2\phi \cos^2(\theta - \alpha)}{\{\sin\delta \sin\phi + \cos\delta \cos\phi \cos(\theta - \alpha)\}^2}.$$

If we assume m, n, μ, ν (m and n representing the same quantities as Bessel's N and n), such that

$$\cos n = \cos\phi \sin(\theta - \alpha)$$
$$\sin n \sin m = \cos\phi \cos(\theta - \alpha)$$
$$\sin n \cos m = \sin\phi,$$

and

$$\cos\nu = \sin\phi \sin(\theta - \alpha)$$
$$\sin\nu \cos\mu = \sin\phi \cos(\theta - \alpha)$$
$$\sin\nu \sin\mu = \cos\phi,$$

and substitute in the equations just obtained we find

$$\frac{d\chi}{d\alpha} = k \cdot \frac{\sin^2\nu \sin\mu \sin(\mu + \delta)}{\cos\delta \sin^2 n \sin^2(m + \delta)},$$

$$\frac{d\chi}{d\delta} = k \cdot \frac{\cot n \cos(m + 2\delta)}{\cot^2\delta \sin^2(m + \delta)},$$

$$\frac{d\psi}{d\alpha} = k \cdot \frac{\cot n \cos m}{\sin^2(m + \delta)},$$

and

$$\frac{d\psi}{d\delta} = k \cdot \frac{1}{\sin^2(m + \delta)}.$$

We have thus the following equations for determining the refraction :—

$$\tan m = \cot\phi \cos(\theta - \alpha), \quad \cot n = \sin m \tan(\theta - \alpha),$$
$$\cot\mu = \tan\phi \cos(\theta - \alpha), \quad \cot\nu = \cos\mu \tan(\theta - \alpha),$$

$$\text{correction to } \Delta\alpha = k \left\{ \frac{\sin\mu \sin^2\nu \sin(\mu + \delta)}{\cos\delta \sin^2 n \sin^2(m + \delta)} \Delta\alpha + \frac{\cot n \cos(m + 2\delta)}{\cos^2\delta \sin^2(m + \delta)} \Delta\delta \right\},$$

and

$$\text{correction to } \Delta\delta = k \left\{ \frac{\cot n \cos m}{\sin^2(m + \delta)} \Delta\alpha + \frac{1}{\sin^2(m + \delta)} \Delta\delta \right\}.$$

To recapitulate we have, therefore, the following formulæ for the adjust-

ment of the plate and the determination of the differences of R. A. and declination :—

Correction to the reading of the position circle $\Delta p = \tan^{-1} \frac{V' - V}{H' - H}$. (α)

Readings of the centre,
$$\begin{cases} H_0 = \frac{1}{2}(H_A + H_B). & (\beta) \\ V_0 = \frac{1}{2}(V_A + V_B) + \frac{1}{8}\frac{(H_B - H_A)^2}{r} \tan \delta. & (\gamma) \end{cases}$$

Co-ordinates of a star,
$$\begin{cases} x = H - H_0. & (\delta) \\ y = V - V_0. & (\epsilon) \end{cases}$$

Then,

$$\alpha_1 - \alpha = \frac{x}{r} \sec \delta + \frac{xy}{r^2} \sec \delta \tan \delta - \tfrac{1}{3}\frac{x^3}{r^3} \sec^3\delta + \frac{xy^2}{r^3} \sec \delta \tan^2\delta. \quad (\zeta)$$

$$\delta_1 - \delta = \frac{y}{r} - \tfrac{1}{2}\frac{x^2}{r^2} \tan \delta - \tfrac{1}{2}\frac{x^2 y}{r^3} \sec^2\delta - \tfrac{1}{3}\frac{y^3}{r^3}, \quad (\eta)$$

in which α and δ represent the R. A. and declination of the adopted centre.

If, then, $\Delta\alpha$, $\Delta\delta$ denote the differences measured from any fundamental star, we have the correction for refraction

$$\text{to } \Delta\alpha = k \left\{ \frac{\sin \mu \sin^2\nu \sin(\mu + \delta)}{\cos \delta \sin^2 n \sin^2(m + \delta)} \Delta\alpha + \frac{\cot n \cos(m + 2\delta)}{\cos^2\delta \sin^2(m + \delta)} \Delta\delta \right\}, \quad (\theta)$$

and,

$$\text{to } \Delta\delta = k \left\{ \frac{\cot n \cos m}{\sin^2(m + \delta)} \Delta\alpha + \frac{1}{\sin^2(m + \delta)} \Delta\delta \right\}. \quad (\iota)$$

And the corrections for reduction

$$\text{to } \Delta\alpha = -[g \cos(G + \alpha) \tan \delta + h \cos(H + \alpha) \sec \delta] \sin 1'' \Delta\alpha$$
$$- [g \sin(G + \alpha) \sec^2\delta + h \sin(H + \alpha) \sec \delta \tan \delta] \sin 1'' \Delta\delta, \quad (\kappa)$$

and

$$\text{to } \Delta\delta = [g \sin(G + \alpha) + h \sin(H + \alpha) \sin \delta] \sin 1'' \Delta\alpha$$
$$+ [i \sin \delta - h \cos(H + \alpha) \cos \delta] \sin 1'' \Delta\delta. \quad (\lambda)$$

Up to this we have supposed that the value of r is known, and that x, y, and r are expressed in terms of the same units. It will, however, be necessary to determine r separately for each plate, and to reduce the values of x and y, which are measured on different screws to a uniform system. For this purpose we have found it convenient to proceed as follows:—

Let H and V be the co-ordinates of a star as measured from the centre on the two screws, and let R_H and R_V be two factors such that $R_H H$, $R_V V$ are equal, respectively, to the number of seconds in the angles whose circular measures are $\frac{x}{r}$ and $\frac{y}{r}$. Hence, $R_H H \sin 1'' = \frac{x}{r}$, and $R_V V \sin 1'' = \frac{y}{r}$. We thus find, if $\Delta\alpha$ is expressed in seconds of arc,

$$\Delta\alpha = R_H H \sec\delta \left[1 + R_V V \sin 1'' \tan\delta + R_V^2 V^2 \sin^2 1'' \tan^2\delta - \tfrac{1}{3} R_H^2 H^2 \sin^2 1'' \sec^2\delta\right].$$

But since the two screws are very nearly alike, we may put

$$R_H = R(1 + \lambda_H), \quad \text{and} \quad R_V = R(1 + \lambda_V),$$

where λ_H and λ_V are small quantities whose squares may be neglected.

We thus find

$$\Delta\alpha = RH(1+\lambda_H)\sec\delta\left[1 + R(1+\lambda_V)\, V \sin 1'' \tan\delta + R^2(1+2\lambda_V)V^2 \sin^2 1'' \tan^2\delta - \tfrac{1}{3} R^2(1+2\lambda_H) H^2 \sin^2 1'' \sec^2\delta\right],$$

or

$$\Delta\alpha = RH\sec\delta\left[1 + RV\sin 1'' \tan\delta + R^2V^2\sin^2 1'' \tan^2\delta - \tfrac{1}{3} R^2H^2 \sin^2 1'' \sec^2\delta\right] + RH\sec\delta\left[1 + RV\sin 1'' \tan\delta + R^2V^2\sin^2 1'' \tan^2\delta - R^2H^2\sin^2 1'' \sec^2\delta\right]\lambda_H + RH\sec\delta\left[RV\sin 1'' \tan\delta + 2R^2V^2\sin^2 1'' \tan^2\delta\right]\lambda_V.$$

Again, if the plate be turned through a right angle so that the screws are interchanged, we shall have

$$R_V V' \sin 1'' = \frac{x}{r}, \quad \text{and} \quad R_H H' \sin 1'' = \frac{y}{r},$$

and we shall obtain, in a similar manner,

$$\begin{aligned}\Delta\alpha = {} & RV' \sec\delta\,[1 + RH' \sin 1'' \tan\delta + R^2H'^2 \sin^2 1'' \tan^2\delta \\ & \qquad - \tfrac{1}{3} R^2V'^2 \sin^2 1'' \sec^2\delta] \\ & + RV' \sec\delta\,[RH' \sin 1'' \tan\delta + 2R^2H'^2 \sin^2 1'' \tan^2\delta]\,\lambda_H, \\ & + RV' \sec\delta\,[1 + RH' \sin 1'' \tan\delta + R^2H'^2 \sin^2 1'' \tan^2\delta \\ & \qquad - R^2V'^2 \sin^2 1'' \sec^2\delta]\,\lambda_V.\end{aligned}$$

And, finally, if by H_1, V_1, H'_1, V'_1 we denote similar measures made on another star, we shall find

$$\begin{aligned}\alpha_1 - \alpha = {} & \sec\delta\,[R(H_1 - H) + R^2(H_1V_1 - HV) \sin 1'' \tan\delta \\ & + R^3(H_1V_1^2 - HV^2) \sin^2 1'' \tan^2\delta - \tfrac{1}{3} R^3(H_1^3 - H^3) \sin^2 1'' \sec^2\delta] \\ & + \sec\delta\,[R(H_1 - H) + R^2(H_1V_1 - HV) \sin 1'' \tan\delta \\ & + R^3(H_1V_1^2 - HV^2) \sin^2 1'' \tan^2\delta - R^3(H_1^3 - H^3) \sin^2 1'' \sec^2\delta]\,\lambda_H \\ & + \sec\delta\,[R^2(H_1V_1 - HV) \sin 1'' \tan\delta \\ & \qquad + 2R^3(H_1V_1^2 - HV^2) \sin^2 1'' \tan^2\delta]\,\lambda_V. \qquad (\mu)\end{aligned}$$

And

$$\begin{aligned}\alpha_1 - \alpha = {} & \sec\delta\,[R(V'_1 - V') + R^2(H'_1V'_1 - H'V') \sin 1'' \tan\delta \\ & + R^3(H'^2_1V'_1 - H'^2V') \sin^2 1'' \tan^2\delta - \tfrac{1}{3} R^3(V'^3_1 - V'^3) \sin^2 1'' \sec^2\delta] \\ & + \sec\delta\,[R^2(H'_1V'_1 - H'V') \sin 1'' \tan\delta \\ & \qquad + 2R^3(H'^2_1V'_1 - H'^2V') \sin^2 1'' \tan^2\delta]\,\lambda_H \\ & + \sec\delta\,[R(V'_1 - V') + R^2(H'_1V'_1 - H'V') \sin 1'' \tan\delta \\ & + R^3(H'^2_1V'_1 - H'^2V') \sin^2 1'' \tan^2\delta - R^3(V'^3_1 - V'^3) \sin^2 1'' \sec^2\delta]\,\lambda_V. \qquad (\nu)\end{aligned}$$

From either of these equations the value of R may be found by putting λ_H and λ_V equal to zero. Substituting this value for R then throughout, the values of λ_H and λ_V may be derived. In general, however, the value of R will be known with sufficient accuracy, and may be substituted at once, and if the two stars are chosen so that they are nearly on the same

parallel the solution will be greatly simplified, for, in that case, the coefficient of λ_V in the first equation given above will be so small that this term may be neglected, while in the second equation that of λ_H need not be taken into account. We have thus the means of determining the value of a revolution of each screw from the transit of a pair of stars.

Having thus determined the values of R_H and R_V, equations (ζ) and (η) become

$$\alpha' - \alpha = R_H H \sec\delta \left[1 + R_V V \sin 1'' \tan\delta - \tfrac{1}{3}(R_H H)^2 \sin^2 1'' \sec^2\delta + (R_V V)^2 \sin^2 1'' \tan^2\delta\right],$$

and

$$\delta' - \delta = R_V V - \tfrac{1}{2}(R_H H)^2 \sin 1'' \tan\delta - \tfrac{1}{2}(R_H H)^2 (R_V V) \sin^2 1'' \sec^2\delta - \tfrac{1}{3}(R_V V)^3 \sin^2 1'',$$

which, if we write X and Y for $R_H H$ and $R_V V$, respectively, assume the form

$$\alpha' - \alpha = AX + BXY + CX^3 + DXY^2, \qquad (\xi)$$

and

$$\delta' - \delta = Y + B'X^2 + C'X^2Y + D'Y^3. \qquad (o)$$

We have thus derived all the formulæ necessary for the deduction of differences of R.A. and declination from measures of a photographic plate in two directions at right angles to each other.

THE CLUSTER IN PERSEUS.

As an application of the method we have taken a photograph of the clusters in Perseus (h and χ Persei) obtained on the 1st of September, 1891. This photograph contains two exposures of 10^m each, the clock having been stopped in the interval for about 1^m. The sidereal times corresponding to the middle of each exposure were $23^h\ 24^m$ and $23^h\ 36^m$, respectively, and the reading of the thermometer 52°·5 F. The star images in the first exposure are slightly elongated owing to the inequalities in the running of the driving clock, but in the second exposure the images appear sensibly circular under the microscope, and are susceptible of very accurate

measurement. We have, accordingly, in what follows, confined ourselves to the second exposure for the determination of the stars' co-ordinates, using the first only for the adjustment of the plate and the determination of the centre, in which the elongation of the images will introduce no loss of accuracy.

Before commencing to measure the plate it was, of course, necessary to examine the irregularities of the screws. For this purpose we ruled, on a plate of glass, two fine lines separated by a space approximately equal to five revolutions of the screw. In order to eliminate any periodic error that might exist, each complete measure consists of five, in which the starting points differ by one-fifth of a revolution. Thus, the measures made, beginning at 023000, 024000, 025000, 026000, and 027000, are all combined to give the measure corresponding to 025000. Treating the measures in the usual way, and denoting by R the reading of the screw, and by f_H the corresponding correction, we obtain the following Table for the correction to the readings of the H screw in which a unit represents $\frac{1}{100,000}$th of an inch, which corresponds approximately to the $\frac{1}{60}$th of a second of arc.

Table I.—Irregularities of the H Screw.

R	f_H	R	f_H	R	f_H	R	f_H
000000	+ 0	050000	+ 5	100000	0	150000	+ 3
005000	1	055000	6	105000	+ 1	155000	− 3
010000	2	060000	7	110000	2	160000	10
015000	4	065000	8	115000	2	165000	16
020000	6	070000	10	120000	3	170000	23
025000	7	075000	11	125000	4	175000	29
030000	7	080000	9	130000	4	180000	23
035000	6	085000	7	135000	4	185000	17
040000	6	090000	4	140000	3	190000	12
045000	5	095000	+ 2	145000	3	195000	− 6
050000	+ 5	100000	0	150000	+ 3	200000	0

In an exactly similar manner we obtain the following Table for the correction to the V screw :—

TABLE II.—IRREGULARITIES OF THE V SCREW.

R	f_V	R	f_V	R	f_V	R	f_V
000000	0	050000	− 32	100000	− 72	150000	− 40
005000	− 5	055000	40	105000	69	155000	44
010000	9	060000	47	110000	66	160000	47
015000	13	065000	54	115000	62	165000	47
020000	17	070000	60	120000	59	170000	46
025000	20	075000	64	125000	56	175000	43
030000	21	080000	68	130000	51	180000	31
035000	22	085000	70	135000	47	185000	20
040000	25	090000	72	140000	44	190000	11
045000	29	095000	72	145000	42	195000	− 5
050000	− 32	100000	− 72	150000	− 40	200000	0

It appears, therefore, that while the errors of the H screw are remarkably small, so that except for the portion lying between 155000 and 195000, they might almost be neglected, those of the V screw are much more considerable, amounting, at the middle of its range, to more than a second of arc. In what follows, however, the corrections from these Tables have been applied to every measure made with the instrument.

For determining the periodic errors two spaces, equal respectively to a half and a quarter of a revolution, were measured on both screws over ten complete revolutions, the starting points being so arranged that each was a tenth of a revolution beyond the previous one. From a discussion of these measures we found that the correction (c) to the H screw is represented by the expression

$$c = + 2{\cdot}91 \cos(u - 52^\circ\ 58') - 1{\cdot}78 \cos(2u - 28^\circ\ 28'),$$

and that the correction (c') to the V screw is represented by the expression

$$c' = - 9{\cdot}66 \cos(u - 56^\circ\ 25') - 1{\cdot}88 \cos(2u - 43^\circ\ 19').$$

In neither case, therefore, does the periodic error amount to 0″·2, and in both the principal part is contained in the first term. This error will, therefore, be eliminated by taking the mean of two observations so that the readings differ by half a revolution of the screw. In the observations given below, the readings made on each occasion were arranged in two series between which the plate was displaced by an amount equal to half a revolution of the screw in both directions, which method of arranging the observations has the double advantage that each series maintains an independent check on the other sufficient to detect any large accidental error of reading, and that the periodic error is at the same time eliminated.

As an example of the process of adjusting the plate and determining the centre, we may take the measures made on 1st December, 1891, corresponding to Sheet 4 in Table V., the letters denoting the same quantities as in equations (α), (β), and (γ), given above.

We find

	H	V	H'	V'
	071423	106017	104145	106021
	—	011	—	026
	—	008	—	022
	—	013	—	021
	—	013	—	012
	—	012	—	019
	—	007	—	027
	—	010	—	019
	—	011	—	017
	071413	010	104163	017
Means,	071418	106011·2	104154	106020·1

We find, accordingly,

$$V' - V = +\,8{\cdot}9 \quad \text{and} \quad H' - H = 32736.$$

Hence, $\Delta p = -\,56''$. (The sign of this correction depends on the direction

in which the graduations of the circle run.) When the plate is turned through this angle we obtain

	H_A	V_A	H_B	V_B
	071412	106030	104143	106030
	424	028	149	029
Means,	071418	106029	104146	106029

Applying the correction f_H and f_V, substituting in equations (β) and (γ), and remarking that $\frac{1}{8} \frac{(H_B - H_A)^2}{r} \tan \delta = 17$, we find,

$$H_0 = 087787 \quad \text{and} \quad V_0 = 105978.$$

These measures were made on the star No. 5 in the Catalogue, this star having been placed in the manner described above, so that its two images were placed nearly symmetrically with regard to the centre.

For determining the screw-values R_H and R_V, the distance of this star from No. 195 in the Catalogue, was measured with the following results:—

	H	V		H	V
5A	071157	106188	5B	103906	106188
	149	190		896	187
195	187548	106056			
	562	056			
	545	068			
	548	056			

And, again,

5A	071147	106185
	143	183

Having moved the plate through a distance equal to half a revolution in

both directions, on the stage of the microscope, by means of the opposing screws, the following readings were obtained :—

	H	*V*	*H*	*V*
5*A*	068680	103669	101429	103650
	685	660	424	665
195	185062	103516		
	046	528		
	055	515		
	051	501		

and

5*A*	068687	103646		
	674	650		

We thus find the quantities H, V, H_1, and V_1, of equation (μ), having corrected for the irregularities of the screws by applying the correction from Table I. and Table II.,

$$H = -\ 016371 \qquad H_1 = +\ 100005$$
$$V = -\ 000018 \qquad V_1 = -\ 000145.$$

Having turned the plate through 90°, the following results were obtained in a similar manner :—

	H	*V*		*H*	*V*
5*A*	100198	128452	5*B*	100201	095803
	204	450		197	797
195	100111	012166			
	106	176			
	101	156			
	106	167			

and

5*A*	100206	128437
	196	447

And having diminished the readings of each screw by half a revolution, we find

	H	V		H	V
5*A*	097709	125948	5*B*	097717	093265
	715	956		704	265
195	097607	009666			
	613	647			
	602	640			
	614	645			

and

5*A*	097711	125940
	707	946

From these we obtain the quantities H', V', and H'_1, V'_1, of equation (ν), viz.:—

$$H' = -\,000016 \qquad H'_1 = -\,000111$$
$$V' = -\,016331 \qquad V'_1 = +\,099907.$$

The difference in R.A. of the two stars was determined on 22nd November, 1891, by sixteen transits across the vertical wires in the South Equatorial. For this purpose the rate of the clock was found by meridian observations of standard stars immediately before and after the observations with the equatorial, all the transits being recorded on the chronograph. The rate of the clock proved to be insensible in the interval of $2\frac{1}{2}$ hours between the first and last observation. The difference in R.A. between No. 5 and No. 195 was thus found to be, after correcting for the effects of refraction,

$$234^s{\cdot}918 \pm 0^s{\cdot}041.$$

The reduction to the beginning of the year, as computed from formula (κ), is $-\,0^s{\cdot}042$, so that we find as the mean difference in R.A. between Nos. 5 and 195,

$$234^s{\cdot}876.$$

This has again to be corrected for refraction and reduction to bring up to

the date of the photograph, viz. to $23^h\ 36^m$ Sidereal Time on the 1st September. We find from the formulæ (θ) and (κ), putting $\Delta\alpha = 0$ in (θ),

$$\begin{aligned} \text{Refraction} &= -\ 0^s{\cdot}070 \\ \text{Reduction} &= -\ 0\ {\cdot}016 \\ \text{Mean } (\alpha' - \alpha) &= 234\ {\cdot}876 \end{aligned}$$

Hence, $\quad \text{apparent } (\alpha' - \alpha) = 234\ {\cdot}790 = 3521''{\cdot}85.$

Substituting this value of $\alpha' - \alpha$, and those already found for H, V, H', V', &c., in equations (μ) and (ν), and remarking that the term containing λ_V in the former, and that containing λ_H in the latter may be neglected, and that R may be taken as being 0·01668, we find

$$R\lambda_H = -\ 0{\cdot}0000163$$

and $\quad R\lambda_V = +\ 0{\cdot}0000004.$

Hence we have, finally,

$$R_H = 0{\cdot}0166637$$

and $\quad R_V = 0{\cdot}0166804.$

These are the values of the screws at the temperature at which the measures were made, and if measures were taken at other temperatures, it would be necessary to investigate the temperature correction to these values. The necessity for this correction is, however, obviated by keeping the temperature of the microscope nearly constant. In the measures given below no such correction has been required, the temperature of the instrument being kept constant within a few degrees.

In order to facilitate the reductions we have computed Tables from equations (θ), (ι), (κ), and (λ). For this purpose we have adopted the R.A. and declination of No. 5, as given by Pihl, in his Memoir entitled "The stellar cluster χ Persei micrometrically surveyed," as his place agrees very closely with that determined by Krüger in his survey of the preceding cluster, *h* Persei, and in his zone-observations. We have thus, for 1891,

$$\begin{aligned} \alpha &= \quad 2^h\ 12^m\ 35^s{\cdot}76. \\ \delta &= +\ 56^\circ\ 35'\ 45''{\cdot}6. \end{aligned}$$

Also,

$$\theta = \quad 23^h\ 36^m \quad \text{and} \quad \phi = 53^\circ\ 23'\ 13''.$$

Hence we find

$$m = 29^\circ\ 55'\cdot0 \qquad \mu = 43^\circ\ 49'\cdot0.$$
$$n = 112\ \ 10\cdot5 \qquad \nu = 120\ \ 31\cdot3.$$

We also find $\log k = 6\cdot4423$ from Bessel's Refraction Tables, from which we obtain the

$$\text{correction to } \Delta\alpha = +\ 0\cdot0002975\ \Delta\alpha + 0\cdot0002989\ \Delta\delta,$$

and

$$\text{correction to } \Delta\delta = -\ 0\cdot0000982\ \Delta\alpha + 0\cdot0002779\ \Delta\delta.$$

Writing these in the form,

$$\text{correction to } \Delta\alpha = \Delta_{\alpha\alpha} + \Delta_{\alpha\delta}$$

and,

$$\text{correction to } \Delta\delta = \Delta_{\delta\alpha} + \Delta_{\delta\delta}$$

we obtain the following Table:—

TABLE III.—REFRACTION.

$\Delta_{\alpha\alpha}$	$\Delta_{\delta\alpha}$	$\Delta\alpha$ or $\Delta\delta$	$\Delta_{\alpha\delta}$	$\Delta_{\delta\delta}$	$\Delta_{\alpha\alpha}$	$\Delta_{\delta\alpha}$	$\Delta\alpha$ or $\Delta\delta$	$\Delta_{\alpha\delta}$	$\Delta_{\delta\delta}$
+ 0″·030	− 0″·010	+ 100″	+ 0″·030	+ 0″·028	+ 0″·773	− 0″·255	+2600″	+ 0″·777	+ 0″·723
·060	·020	200	·060	·056	·803	·265	2700	·807	·750
·089	·029	300	·090	·083	·833	·275	2800	·837	·778
·119	·039	400	·120	·111	·863	·285	2900	·867	·806
·149	·049	500	·149	·139	·892	·295	3000	+ 0·897	+ 0·834
0·179	0·059	600	0·179	0·167	0·922	0·304	3100	—	—
·208	·069	700	·209	·195	·952	·314	3200	—	—
·238	·079	800	·239	·222	0·981	·324	3300	—	—
·268	·088	900	·269	·250	1·011	·334	3400	—	—
·297	·098	1000	·299	·278	·041	·344	3500	—	—
0·327	0·108	1100	0·329	0·306	1·071	0·354	3600	—	—
·357	·118	1200	·359	·333	·101	·363	3700	—	—
·387	·128	1300	·389	·361	·130	·373	3800	—	—
·416	·137	1400	·418	·389	·160	·383	3900	—	—
·446	·147	1500	·448	·417	·190	·393	4000	—	—
0·476	0·157	1600	0·478	0·445	1·220	0·403	4100	—	—
·506	·167	1700	·508	·472	·249	·412	4200	—	—
·535	·177	1800	·538	·500	·279	·422	4300	—	—
·565	·187	1900	·568	·528	·309	·432	4400	—	—
·595	·196	2000	·598	·556	·339	·442	4500	—	—
0·625	0·206	2100	0·628	0·584	1·368	0·452	4600	—	—
·654	·216	2200	·658	·611	·398	·462	4700	—	—
·684	·226	2300	·687	·639	·428	·471	4800	—	—
·714	·236	2400	·717	·667	·458	·481	4900	—	—
+ 0·744	− 0·245	+ 2500	+ 0·747	+ 0·695	+ 1·488	− 0·491	+ 5000	—	—

This Table is entered with the argument $\Delta\alpha$ or $\Delta\delta$ in the middle column, and the corresponding quantities are found in one or other of the remaining columns with a sign corresponding to a positive argument. If the argument is negative the quantities $\Delta_{\alpha\alpha}$, &c., must be taken with a contrary sign.

To compute the reduction Table we have, from the "Nautical Almanac" for 1st September, 1891,

$$\log g = +\,0{\cdot}9699 \qquad G = 327^{\circ}\ 23',$$

$$\log h = +\,1{\cdot}2782 \qquad H = 112\ \ 29,$$

$$\log i = +\,0{\cdot}8813.$$

Hence we find

$$\text{correction to } \Delta\alpha = +\,0{\cdot}0000693\ \Delta\alpha - 0{\cdot}0001444\ \Delta\delta,$$

and

$$\text{correction to } \Delta\delta = +\,0{\cdot}0000438\ \Delta\alpha + 0{\cdot}0000726\ \Delta\delta.$$

Writing these, as in the case of the refraction, in the form,

$$\text{correction to } \Delta\alpha = \Delta'_{\alpha\alpha} + \Delta'_{\alpha\delta},$$

$$\text{correction to } \Delta\delta = \Delta'_{\delta\alpha} + \Delta'_{\delta\delta},$$

we find Table IV., from which the corrections for reduction can at once be obtained.

TABLE IV.—REDUCTION.

$\Delta'_{\alpha\alpha}$	$\Delta'_{\delta\alpha}$	$\Delta\alpha$ or $\Delta\delta$	$\Delta'_{\alpha\delta}$	$\Delta'_{\delta\delta}$	$\Delta'_{\alpha\alpha}$	$\Delta'_{\delta\alpha}$	$\Delta\alpha$ or $\Delta\delta$	$\Delta'_{\alpha\delta}$	$\Delta'_{\delta\delta}$
+ 0·″007	+ 0·″004	+ 100″	− 0·″014	+ 0·″007	+ 0·″076	+ 0·″048	+ 1100″	− 0·″159	+ 0·″080
·014	·009	200	·029	·015	·083	·053	1200	·173	·087
·021	·013	300	·043	·022	·090	·057	1300	·188	·094
·028	·018	400	·058	·029	·097	·061	1400	·202	·102
·035	·022	500	·070	·036	·104	·066	1500	·217	·109
0·042	0·026	600	0·087	0·044	0·111	0·070	1600	0·231	0·116
·049	·031	700	·101	·051	·118	·074	1700	·245	·123
·055	·035	800	·116	·058	·125	·079	1800	·260	·131
·062	·039	900	·130	·065	·132	·083	1900	·274	·138
+ 0·069	+ 0·044	+ 1000	− 0·144	+ 0·073	+ 0·139	+ 0·088	+ 2000	− 0·289	+ 0·145

TABLE IV.—REDUCTION—*continued.*

$\Delta'_{\alpha\alpha}$	$\Delta'_{\delta\alpha}$	$\Delta\alpha$ or $\Delta\delta$	$\Delta'_{\alpha\delta}$	$\Delta'_{\delta\delta}$	$\Delta'_{\alpha\alpha}$	$\Delta'_{\delta\alpha}$	$\Delta\alpha$ or $\Delta\delta$	$\Delta'_{\alpha\delta}$	$\Delta'_{\delta\delta}$
+ 0·146	+ 0·092	+2100″	− 0·303	+ 0·152	+ 0·249	+ 0·158	+ 3600″	—	—
·152	·096	2200	·318	·160	·256	·162	3700	—	—
·159	·101	2300	·332	·167	·263	·166	3800	—	—
·166	·105	2400	·347	·174	·270	·171	3900	—	—
·173	·110	2500	·361	·181	·277	·175	4000	—	—
0·180	0·114	2600	0·375	0·189	0·284	0·180	4100	—	—
·187	·118	2700	·390	·196	·291	·184	4200	—	—
·194	·123	2800	·404	·203	·298	·188	4300	—	—
·201	·127	2900	·419	·211	·305	·193	4400	—	—
·208	·131	3000	− 0·433	+ 0·218	·312	·197	4500	—	—
0·215	0·136	3100	—	—	0·319	0·201	4600	—	—
·222	·140	3200	—	—	·326	·206	4700	—	—
·229	·145	3300	—	—	·333	·210	4800	—	—
·236	·149	3400	—	—	·340	·215	4900	—	—
+ 0·243	+ 0·153	+3500	—	—	+ 0·346	+ 0·219	+5000	—	—

It only remains to compute the values of the coefficients in equations (ξ) and (o). These are found to be

$$\log A = + 0{\cdot}2592119,$$
$$\log B = + 5{\cdot}12559,$$
$$\log C = - 9{\cdot}672,$$
$$\log D = + 9{\cdot}992,$$

and

$$\log B' = - 4{\cdot}56534,$$
$$\log C' = - 9{\cdot}589,$$
$$\log D' = - 8{\cdot}894.$$

Having then determined the readings of the centre and of the stars in the two positions of the plate in the manner described above, the computation can be very easily conducted on ruled sheets, as shown on opposite page. These sheets are ruled into fourteen columns, each of which contains the whole computation relating to a single star, but only two of these columns are given as specimens, and these are divided into two portions, in order to bring the whole form within the limits of the page.

FORM FOR COMPUTING $\Delta\alpha$ AND $\Delta\delta$.

1.	Star,	5	200
2.	H_1,	056313	176349
3.	H_2,	058830	178818
4.	$f_{H_1} + H_1$,	056319	176322
5.	$f_{H_2} + H_2$,	058837	178794
6.	$f_{H_1} + H_1 - H_0$,	− 016380	103623
7.	$f_{H_2} + H_2 - H_0'$,	− 016345	103612
8.	H,	− 016363	+ 103618
9.	V_1,	103447	144110
10.	V_2,	105939	146603
11.	$f_{V_1} + V_1$,	103377	144068
12.	$f_{V_2} + V_2$,	105871	146562
13.	$V_0 - f_{V_1} - V_1$,	+ 000017	− 040674
14.	$V_0' - f_{V_2} - V_2$,	+ 000020	− 040671
15.	V,	+ 000019	− 040673
16.	$R_H H = X$,	− 272·668	+ 1726·660
17.	log X,	2·4356344	3·2372067
18.	log Y,	9·50095	2·83150
19.	log XY,	1·93658	6·06871
20.	log X^2,	4·87127	6·47441
21.	log X^3,	7·307	9·712
22.	log X^2Y,	4·372	9·306
23.	log XY^2,	1·437	8·900
24.	log Y^3,	8·503	8·494
25.	log AX,	2·6948463	3·4964186
26.	log BXY,	7·06217	1·19430
27.	log CX^3,	6·979	9·384
28.	log DXY^2,	1·429	8·892

29.	AX,	− 495·27	+ 3136·31
30.	BXY,	. . .	− 15·64
31.	CX^3,	. . .	− ·24
32.	DXY^2,	. . .	+ ·08
33.	Apparent $\Delta\alpha$ from centre,	− 495·27	+ 3120·51
34.	Apparent $\Delta\alpha$ from No. 5,	—	+ 3615·78
35.	Table III. Refraction $\Delta_{\alpha\alpha}$,	—	+ 1·08
36.	Table III. ,, $\Delta_{\alpha\delta}$,	—	− ·21
37.	Table IV. Reduction $\Delta'_{\alpha\alpha}$,	—	+ ·25
38.	Table IV. ,, $\Delta'_{\alpha\delta}$,	—	+ ·10
39.	Mean $\Delta\alpha$ from No. 5 (arc.),	—	+ 3617·00
40.	,, ,, ,, ,, (time),	—	+ 241·133
41.	log $B'X^2$,	9·43661	1·03975
42.	log $C'X^2Y$,	3·961	8·895
43.	log $D'Y^3$,	. . .	7·388
44.	$R_V V = Y$,	+ 0·32	− 678·44
45.	$B'X^2$,	− 0·27	− 10·95
46.	$C'X^2Y$,	. . .	+ ·08
47.	$D'Y^3$,	. . .	. . .
48.	Apparent $\Delta\delta$ from centre,	+ 0·05	− 689·31
49.	Apparent $\Delta\delta$ from No. 5,	—	− 689·36
50.	Table III. Refraction $\Delta_{\delta\alpha}$,	—	− ·35
51.	Table III. ,, $\Delta_{\delta\delta}$,	—	− ·19
52.	Table IV. Reduction $\Delta'_{\delta\alpha}$,	—	+ ·16
53.	Table IV. ,, $\Delta'_{\delta\delta}$,	—	− ·05
54.	Mean $\Delta\delta$ from No. 5,	—	− 689·79

On the 1st line is given the current number of the star. On the 2nd and 3rd lines are found the readings (H_1 and H_2) of the horizontal screw, and on the 9th and 10th the readings (V_1 and V_2) of the other. Correcting, these, for the irregularities of the screws from Tables I. and II., we find the quantities in the 4th and 5th, 11th and 12th lines, respectively. We then subtract H_0 from $f_{H_1} + H_1$ and H'_0 from $f_{H_2} + H_2$, entering the results in the 6th and 7th lines respectively, the mean of which (H) is entered in the 8th line.

We then subtract $f_{V_1} + V_1$ and $f_{V_2} + V_2$ from V_0 and V'_0, respectively, writing the results in the 13th and 14th lines, and enter the mean (V) in the 15th line. Multiplying H by R_H, we obtain X in the 16th line, and V by R_V, which is equal to Y, is entered in the 44th line. We then enter $\log X$ and $\log Y$ in the 17th and 18th lines, and find $\log XY$, $\log X^2$, $\log X^3$, $\log X^2Y$, $\log XY^2$, and $\log Y^3$ in the 19th, 20th, 21st, 22nd, 23rd, and 24th lines. We then compute $\log AX$, $\log BXY$, $\log CX^3$, $\log DXY^2$, with the values of the constants found above, in the 25th, 26th, 27th, and 28th lines, and enter AX, BXY, CX^3, and DXY^2 in the four next lines. Adding these together, we find the "apparent $\Delta\alpha$ from the centre" in the 33rd line. Similarly we compute the logarithms of $B'X^2$, $C'X^2Y$, and $D'Y^3$ in the 41st, 42nd and 43rd lines, and enter these quantities themselves in the 45th, 46th, and 47th lines, the sum of which added to Y is the "apparent $\Delta\delta$ from centre" in the 48th line. The next step is to subtract the value of "apparent $\Delta\alpha$ from centre," which corresponds to No. 5 from all the other quantities in the same line, writing the result as "apparent $\Delta\alpha$ from No. 5" in the 34th line. Similarly, if the quantity in the 48th line corresponding to No. 5 is not zero (as it would be if the observations were altogether free from error), we subtract this from all the other quantities in the same line, and write the result as "apparent $\Delta\delta$ from No. 5" in the 49th line. With "apparent $\Delta\alpha$ from No. 5," then, as argument, we enter Tables III. and IV., and write the corresponding corrections, in the 35th, 37th, 50th, and 52nd lines, and with "apparent $\Delta\delta$ from No. 5," as argument, we find the corrections which are written in the 36th, 38th, 51st, and 53rd lines, respectively. We then add together the quantities in the 34th, 35th, 36th, 37th, and 38th lines. This gives us the mean $\Delta\alpha$ from No. 5 in arc, which,

divided by 15, is entered in the 40th line as "Mean $\Delta\alpha$ from No 5. (time)." In like manner, if we add together the quantities in the 49th, 50th, 51st, 52nd, and 53rd lines, we find the "Mean $\Delta\delta$ from No. 5" in the 54th line.

All the observations given below have been reduced in this way, and the various steps of the computation will be found in Tables V. and VI. The magnitudes of the stars have been taken from Mr. Pihl's Catalogue. The numbers in the first column of Table V., and in the fourth column in Table VI., refer to the sheets on which the computations were conducted, and these two tables contain all the principal steps of the computation, except a small correction to $\Delta\alpha$, of the form $0{\cdot}000531.\ Y$, depending on the fact that the angle between the two screws differs slightly from a right angle. The positions of the stars are given relatively to No. 5, and not reduced to absolute R.A. and declination, as these will depend entirely on the position adopted for the fundamental star. It is, besides, to changes in the relative positions of the stars that we must look for information as to the mechanical condition of the cluster, or with regard to its parallax.

It will possibly be objected that the results given below cannot be accorded any high degree of accuracy, depending, as they do, on the measures of a single plate; and it must be admitted that before they could be looked upon as final, the positions ought to be measured on several plates taken under various conditions. We had intended to take further photographs of the cluster, but pressure of other work, and some necessary alterations in the photographic parts of the telescope prevented our doing so, but it is intended next autumn, when the clusters are in a favourable position for observation, to repeat the investigation.

In the meanwhile the results already obtained are put forward as an instalment, and as an application of the methods of reduction which we have been led to adopt. In this Memoir we have confined ourselves to the problem of deducing differences of R.A. and Declination from the rectangular co-ordinates on the plate. The problem of deducing the distance and position angle of two stars from the polar co-ordinates has been treated by Mr. Harold Jacoby in the "Astronomical Journal," No. 233. In this Paper the author confines his attention to terms of the second degree, whereas the preceding investigation shows that when such large distances as half a degree are measured on the plates, the terms of the third degree may

amount to very nearly half a second of arc. We hope before long to publish formulæ by which these terms may conveniently be retained in reducing the polar co-ordinates to position angle and distance.

The measures of the preceding cluster (*h* Persei) have not yet been completed, and are omitted from this Memoir, as it would have too long delayed its publication to wait for them.

The values of $\Delta\alpha$ and $\Delta\delta$ in the catalogue below are for the date 1891.0, but in order to compare our measures with those of Dr. H. C. Vogel and Mr. Pihl, we have reduced our places to 1870, which is the epoch selected by both these observers. In the 13th, 14th, 15th, and 16th columns of Table VI. are given the differences in R.A. and declination between our measures and theirs, $D - V$ denoting the difference between ours and Dr. Vogel's, and $D - P$ the difference between ours and Mr. Pihl's. The agreement of Mr. Pihl's R.A.s with those of Dr. Vogel is more apparent than real, as the former found it necessary to apply a correction to his measures in order to reconcile them with the Leipzig results.

In bringing the differences of R.A. and declination back to 1870 we have applied the correction for precession in the following way:—

The ordinary precession formulæ are

$$\frac{d\alpha}{dt} = m + n \tan\delta \sin\alpha, \text{ and } \frac{d\delta}{dt} = n \cos\alpha,$$

from which we derive

$$\Delta\frac{d\alpha}{dt} = n \tan\delta \cos\alpha\, \Delta\alpha + n \sec^2\delta \sin\alpha\, \Delta\delta = \Delta''_{\alpha\alpha} + \Delta''_{\alpha\delta},$$

and

$$\Delta\frac{d\delta}{dt} = -n \sin\alpha\, \Delta\alpha = \Delta''_{\delta\alpha},$$

from which we obtain Table VII.

The comparison between the measurements made by us and by Dr. Vogel shows a good accordance in declination, allowing for a constant difference depending on the adopted position of the fundamental star. In the R.A.s, however, there appears to be a small, though systematic, discrepancy between the Leipzig observations and ours. This, probably, arises from the different plans adopted for determining the parallel.

It seems to us that among the advantages of the photographic measures must be included a more strictly defined method of investigating the parallel

than is possible in the older methods of observation. Further investigation on this subject will doubtless be necessary.

TABLE V.—POSITION OF THE ZERO STAR AND THE CENTRE OF THE PLATE.

Sheet.	H_A.	H_B.	V_A.	V_B.	H_0.	V_0.	Apparent $\Delta\alpha$ of No. 5 from Centre.	Apparent $\Delta\delta$ of No. 5 from Centre.
1	068716 071192	101424 103913	103669 106163	103670 106165	085075 087557	103616 106113	− 494″·94	+ 0″·01
2	071380 068897	104102 101621	106005 103500	106005 103497	087746 085264	105954 103446	495·06	0·00
3	068902 071405	101624 104129	103492 105996	103493 106002	085268 087772	103439 105948	495·06	+ 0·03
4	071418 068928	104146 101648	106029 103526	106029 103526	087787 085293	105978 103473	495·09	+ 0·01
5	071435 068943	104164 101676	106069 103606	106068 103604	087805 085315	106018 103552	495·21	0·00
6	068898 071402	101619 104141	103588 106088	103594 106082	085264 087777	103538 106031	− 495·18	+ 0·03
7	068972 071468	101685 104215	103591 106093	103593 106101	085333 087847	103539 106046	495·18	+ 0·05
8	071480 068987	104207 101704	106110 103620	106109 103625	087849 085350	106058 103569	495·06	+ 0·03
9	068843 071357	101583 104085	103731 106221	103730 106220	085218 087731	103678 106170	495·34	+ 0·01
10	071321 068843	104061 101545	106168 103671	106170 103672	087696 085199	106118 103618	495·06	+ 0·03
11	068869 071384	101595 104119	103707 106205	103705 106200	085237 087757	103653 106151	− 495·21	− 0·02
12	071368 068694	104106 101426	106224 103725	106222 103727	087742 085065	106172 103673	495·27	+ 0·01
13	068692 071198	101430 103966	103719 106233	103716 106233	085066 087588	103665 106182	495·55	+ 0·01
14	071198 068698	103966 101430	106233 103740	106233 103739	087588 085069	106182 103686	495·51	+ 0·01
15	056313 058830	089074 091525	103447 105939	103447 105945	072699 075182	103394 105891	495·27	+ 0·05
16	058839 056355	091557 089103	105943 103457	105946 103456	075203 072734	105893 103403	− 495·33	+ 0·01
17	057691 055191	090419 087937	103571 101078	103567 101079	074060 071569	103516 101024	495·40	0·00
18	071149 068681	103901 101426	106186 103656	106187 103657	087530 085058	106135 103602	495·46	+ 0·01
19	071213 068735	103940 101463	106268 103770	106268 103773	087582 085104	106217 103718	495·15	+ 0·01

Table VI.—Catalogue of 223 Stars in the Cluster χ Persei.

No.	No. in Pihl.	No. in Vogel.	Sheet.	*H.*	*V.*	*X.*	*Y.*	Apparent Δα from Centre.	Apparent Δδ from Centre.	Mean Δα from No. 5.	Mean Δδ from No. 5.	Δα		Δδ.		Mag.
												D − *V.*	*D* − *P.*	*D* − *V.*	*D* − *P.*	
										s.	′ ″	*s.*	*s.*	″	″	
1	1	—	1	062216 064708	084327 086814	− 380″699	+ 323″00	− 693″14	+ 322″47	− 13·04	+ 5 22″6	—	+ 0·03	—	− 0″2	10·5
2	2	—	,,	063953 066429	098139 100643	351·887	+ 92·49	639·61	+ 92·03	9·60	+ 1 32·1	—	− 0·06	—	− 1·8	10·6
3	3	—	,,	066895 069384	100776 103228	302·729	+ 48·92	550·08	+ 48·58	3·65	+ 48·6	—	− 0·15	—	− 1·3	10·7
4	4	—	,,	068445 070940	146200 148669	276·851	− 709·40	500·26	− 709·68	− 0·74	− 11 49·9	—	− 0·05	—	− 0·2	11·4
5	5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	8·6
6	6	—	,,	068786 071267	088922 091420	271·268	+ 246·29	493·62	+ 246·02	+ 0·22	+ 4 6·1	—	− 0·07	—	+ 0·6	10·6
7	7	—	,,	072578 075066	092281 094780	208·013	+ 190·26	378·37	+ 190·07	7·88	+ 3 10·1	—	− 0·13	—	− 0·6	10·9
8	8	—	,,	073256 075725	069688 072172	196·865	+ 567·05	359·09	+ 566·91	9·37	+ 9 27·1	—	− 0·04	—	− 0·4	11·1
9	11	—	,,	075499 077996	126380 128826	159·272	− 378·39	288·50	− 378·48	13·56	− 6 18·6	—	− 0·02	—	− 1·1	7·8
10	12	—	,,	076730 079241	057386 059882	138·659	+ 771·89	253·30	+ 771·82	16·53	+ 12 52·1	—	+ 0·03	—	+ 0·3	10·8
11	13	—	,,	077726 080187	078418 080906	122·478	+ 421·50	222·16	+ 421·44	18·41	+ 7 1·6	—	− 0·05	—	+ 1.4	10·9
12	14	—	,,	078128 080605	123149 125658	115·646	− 324·98	209·55	− 325·03	18·86	− 5 25·2	—	+ 0·10	—	− 2·5	10·7
13	17	—	2	082090 079592	059867 057344	94·250	+ 769·62	172·18	+ 769·59	21·95	+ 12 49·8	—	− 0·03	—	+ 0·1	8·9
14	18	—	,,	084320 081827	109428 106947	57·057	− 57·05	103·60	− 57·06	26·08	− 57·1	—	− 0·02	—	− 0·6	11·2
15	19	—	,,	087361 084879	052646 050156	− 6·299	+ 889·62	− 11·51	+ 889·61	32·73	+ 14 49·9	—	− 0·15	—	+ 1·1	11·1
16	20	—	,,	088759 086259	053873 051373	+ 16·814	+ 869·27	+ 30·74	+ 869·26	35·53	+ 14 29·5	—	− 0·24	—	+ 2·1	10·5
17	21	1	,,	089069 086551	078694 076207	21·829	+ 455·63	39·78	+ 455·63	35·91	+ 7 35·7	− 0·11	− 0·06	− 1·3	− 0·3	10·8
18	22	—	,,	090105 087620	143100 140620	39·360	− 619·11	71·16	− 619·12	37·43	− 10 19·4	—	− 0·04	—	− 1·4	8·8
19	23	2	,,	091234 088752	060722 058220	58·190	+ 755·21	106·29	+ 755·20	40·51	+ 12 35·4	− 0·44	− 0·37	− 4·4	− 1·6	10.6
20	24	—	,,	092668 090160	124382 121897	81·852	− 306·62	148·34	− 306·64	42·75	− 5 6·8	—	+ 0·03	—	− 1·4	10·2
21	25	3		092683	099084											

25	29	—	,,	096508 094000	049359 046900	145·307	+ 943·79	280·00	+ 943·76	51·81	+ 15 44·6	—	− 0·41		+ 1·3	10·2
26	30	—	,,	097092 094599	122627 120147	155·672	− 277·38	282·18	− 277·47	51·68	− 4 37·6	—	− 0·09	—	− 1·7	10·4
27	31	—	3	094577 097100	044677 047147	155·306	+ 980·99	284·14	+ 980·89	52·50	+ 16 21·1	—	− 0·28	—	+ 1·2	10·5
28	32	—	,,	096083 098555	164016 166534	179·951	− 1009·73	324·45	− 1009·84	54·10	− 16 50·3	—	− 0·18	—	− 3·1	8·8
,,	,,	—	19	098346 095883	166794 164302	179·501	− 1009·71	323·63	− 1009·82	54·06	− 16 50·2	—	—	—	—	—
29	33	—	3	096429 098950	153455 155979	186·134	− 833·84	336·03	− 833·97	54·98	− 13 54·3	—	− 0·17	—	− 1·8	7·9
30	35	—	,,	097055 099538	156016 158476	196·248	− 875·84	354·19	− 875·97	56·16	− 14 36·3	—	− 0·27	—	− 1·9	8·8
31	34	8	,,	096849 099371	093968 096435	193·132	+ 159·53	351·22	+ 159·39	56·52	+ 2 39·4	− 0·08	− 0·06	− 1·0	− 1·0	9·7
32	36	9	,,	097237 099763	074939 077390	199·631	+ 476·96	363·88	+ 476·81	57·54	+ 7 56·9	− 0·30	− 0·03	− 1·0	− 0·6	10·3
33	37	10	,,	098230 100762	098223 100721	216·228	+ 88·29	393·01	+ 88·12	0 59·28	+ 1 28·1	− 0·16	− 0·01	+ 0·5	+ 0·5	11·5
34	38	11	,,	098694 101230	093146 095625	223·993	+ 173·14	407·38	+ 172·96	1 0·28	+ 2 52·9	− 0·15	− 0·10	− 0·4	− 0·5	9·4
35	39	12	,,	098629 101151	058370 060854	222·794	+ 752·74	406·93	+ 752·56	0·56	+ 12 32·7	+ 0·15	− 0·03	− 1·0	+ 0·5	9·2
36	40	13	,,	099256 101787	106080 108527	233·308	− 42·40	423·65	− 42·60	1·25	− 42·7	− 0·33	0·02	− 0·4	− 1·5	11·2
37	41	—	,,	099051 101602	047473 049967	230·059	+ 934·17	420·77	+ 933·97	1·59	+ 15 34·2	—	− 0·13	—	− 1·0	10·0
38	42	—	3	100997 103531	142330 144807	262·353	− 647·73	474·28	− 647·98	4·30	− 10 48·3	—	− 0·16	—	− 3·6	10·5
,,	,,	—	19	103318 100843	145102 142607	262·253	− 647·93	474·09	− 648·18	4·29	− 10 48·5	—	—	—	—	—
39	43	—	3	101713 104251	146315 148776	274·334	− 714·09	495·69	− 714·37	5·69	− 11 54·7	—	− 0·18	—	− 1·4	10·4
40	44	—	,,	101991 104495	131933 134390	278·684	− 474·04	504·45	− 474·33	6·40	− 7 54·6	—	− 0·12	—	− 1·1	11·1
41	45	—	4	104578 102097	140656 138168	279·917	− 577·84	506·29	− 578·13	6·47	− 9 38·4	—	− 0·15	—	− 3·7	11·2
42	46	19	,,	104854 102360	084194 081726	284·399	+ 364·20	517·96	+ 363·90	7·75	+ 6 4·0	− 0·42	− 0·22	− 1·5	− 0·3	11·5
43	47	20	,,	105270 102773	079187 076723	291·315	+ 447·65	530·90	+ 447·34	8·67	+ 7 27·4	− 0·06	+ 0·04	− 0·8	+ 0·3	10·1
44	48	21	,,	105204 102711	065862 063365	290·248	+ 669·97	529·82	+ 669·66	8·72	+ 11 9·8	− 0·05	− 0·13	0·0	− 0·2	10·7
45	49	25	,,	107378 104876	082060 079567	+ 326·409	+ 400·00	+ 594·64	+ 399·61	+ 1 12·89	+ 6 39·7	− 0·28	− 0·25	+ 0·3	+ 1·6	10·8

TABLE VI.—CATALOGUE OF 223 STARS IN THE CLUSTER χ PERSEI—*continued.*

No.	No. in Pihl.	No. in Vogel.	Sheet.	*H.*	*V.*	*X.*	*Y.*	Apparent Δα from Centre.	Apparent Δδ from Centre.	Mean Δα from No. 5.	Mean Δδ from No. 5.	Δα.		Δδ.		Mag.
												D − *V.*	*D* − *P.*	*D* − *V.*	*D* − *P.*	
										m. *s.*		*s.*	*s.*			
46	50	28	4	107991 105465	080660 078167	+ 336·″423	+ 423·″33	+ 612·″99	+ 422·″91	+ 1 14·12	+ 7 3·″0	− 0·44	− 0·29	+ 0·″7	+ 0·″3	10·9
47	51	29	,,	108351 105850	104577 102110	342·639	+ 24·22	622·48	+ 23·79	14·54	+ 23·7	− 0·21	− 0·08	− 1·6	− 1·2	10·3
48	52	30	,,	108726 106227	078752 076265	348·905	+ 455·09	635·88	+ 454·64	15·67	+ 7 34·7	− 0·13	− 0·09	− 1·2	+ 0·8	10·7
49	53	31	,,	109604 107106	119851 117385	363·535	− 230·72	659·21	− 231·21	16·86	− 3 51·4	+ 0·11	+ 0·06	− 2·3	− 0·8	10·6
50	54	33	,,	109751 107260	062839 060383	366·051	+ 719·96	668·44	+ 719·47	17·98	+ 11 59·6	− 0·11	− 0·11	− 0·3	+ 0·5	9·2
51	55	—	,,	110438 107960	145574 143070	377·616	− 659·78	682·59	− 660·30	18·18	− 11 0·6	—	+ 0·04	—	− 0·9	10·1
52	56	34	,,	110208 107696	057720 055290	373·500	+ 805·03	682·46	+ 804·52	18·97	+ 13 24·7	− 0·12	− 0·22	− 0·3	+ 1·5	10·7
,,	,,	,,	5	110222 107730	057780 055353	373·567	+ 805·00	682·59	+ 804·49	18·99	+ 13 24·7	—	—	—	—	—
53	57	35	4	110854 108341	113714 111268	384·248	− 128·46	697·29	− 129·00	19·45	− 2 9·1	+ 0·26	+ 0·04	− 0·4	+ 0·4	11·2
54	58	—	4	111369 108869	137072 134603	392·947	− 518·18	711·04	− 518·75	20·15	− 8 39·0	—	+ 0·02	—	− 0·9	11·1
55	59	36	5	111011 108527	081533 079113	386·781	+ 409·17	704·67	+ 408·62	20·24	+ 6 48·7	− 0·18	− 0·17	− 3·9	+ 0·2	10·6
56	60	—	,,	111740 109238	044780 042321	398·779	+1021·87	729·81	+1021·27	22·25	+ 17 1·5	—	− 0·06	—	0·0	11·1
57	61	39	,,	113258 110740	060249 057793	423·941	+ 764·11	774·40	+ 763·44	25·09	+ 12 43·6	− 0·06	− 0·10	− 0·4	+ 1·5	9·1
58	62	—	,,	113603 111115	042908 040494	429·940	+1052·70	787·02	+1052·00	26·08	+ 17 32·3	—	− 0·14	—	+ 1·1	11·0
59	63	41	,,	114090 111610	107029 104580	438·122	− 15·86	795·72	− 16·57	26·08	− 16·6	− 0·14	− 0·14	− 1·5	− 0·4	10·5
60	64	42	,,	114209 111730	099688 097265	440·105	+ 106·42	800·04	+ 105·71	26·44	+ 1 45·7	+ 0·03	+ 0·08	− 0·9	− 0·7	9·1
61	65	43	,,	114338 111849	080420 077943	442·171	+ 428·19	805·70	+ 427·47	26·99	+ 7 7·5	− 0·11	− 0·22	+ 2·3	+ 0·1	10·9
62	66	44	,,	114471 111997	087724 085305	444·521	+ 305·94	809·25	+ 305·21	27·16	+ 5 5·2	− 0·20	− 0·21	− 0·5	0·0	10·6
63	67	45	,,	114588 112100	094841 092390	446·354	+ 187·50	811·88	+ 186·77	27·27	+ 3 6·8	− 0·15	− 0·11	− 1·9	− 1·0	10·6
64	68	46	,,	115198 112693	075968 073520	456·369	+ 502·16	832·02	+ 501·39	28·79	+ 8 21·5	− 0·12	− 0·05	0·0	− 0·5	10·7
65	69	—	,,	116021 113521	134185 131740	470·133	− 469·20	851·01	− 470·01	29·53	− 7 50·2	—	− 0·01	—	− 2·0	9·6

				117244	101394											
71	76	55	,,	116163 118673	088410 090877	514·942	+ 253·74	937·07	+ 252·77	35·65	+ 4 12·7	+ 0·01	+ 0·05	− 0·4	− 1·0	9·3
72	77	—	,,	116963 119455	144780 147242	528·089	− 686·97	954·39	− 687·98	36·31	− 11 28·3	—	− 0·04	—	− 1·8	7·9
73	78	56	,,	117098 119612	071818 074295	530·522	+ 530·27	967·40	+ 529·23	37·83	+ 8 49·3	+ 0·08	+ 0·11	+ 0·5	+ 0·9	6·6
74	79	57	,,	117709 120218	102797 105288	540·670	+ 13·53	982·16	+ 12·46	38·53	+ 0 12·3	+ 0·19	− 0·01	− 0·9	0·0	10·2
75	80	—	,,	117409 119940	039648 042100	535·855	+1066·48	981·01	+1065·41	39·03	+ 17 45·7	—	− 0·11	—	+ 1·2	10·7
76	82	59	,,	117742 120303	078225 080692	541·654	+ 423·57	986·92	+ 422·49	39·08	+ 7 2·5	− 0·17	− 0·22	− 1·0	− 0·9	11·4
77	81	58	,,	117580 120099	051000 053470	538·604	+ 877·14	984·66	+ 876·05	39·17	+ 14 36·2	− 0·11	− 0·03	− 0·5	+ 0·8	9·6
78	83	60	,,	117900 120463	079952 082367	544·303	+ 395·21	991·54	+ 394·12	39·36	+ 6 34·1	− 0·18	− 0·05	− 1·4	− 0·1	10·7
79	84	63	,,	118499 121040	090397 092837	554·101	+ 220·83	1008·09	+ 219·70	40·38	+ 3 39·7	− 0·14	− 0·20	− 0·8	− 1·0	10·7
80	85	—	,,	119957 122505	136317 138765	578·464	− 546·47	1046·51	− 547·69	42·52	− 9 8·0	—	+ 0·14	—	− 3·1	10·3
,,	,,	—	19	122307 119826	138950 136479	578·664	− 545·47	1046·89	− 546·69	42·55	− 9 7·0	—	—	—	—	—
81	86	67	6	119987 122541	092962 095442	579·014	+ 177·71	1053·08	+ 176·48	43·35	+ 2 56·4	+ 0·08	+ 0·08	− 0·3	− 0·3	8·3
82	87	68	7	120113 122625	089795 092287	579·597	+ 230·57	1054·55	+ 229·34	43·48	+ 3 49·3	− 0·09	− 0·08	− 0·4	+ 1·2	10·6
83	88	69	,,	120230 122755	097173 099666	581·663	+ 107·50	1057·36	+ 106·26	43·60	+ 1 46·2	+ 0·04	− 0·05	− 1·5	+ 0·4	10·6
84	89	70	,,	120266 122785	094625 097121	582·213	+ 149·97	1058·69	+ 148·72	43·71	+ 2 28·6	+ 0·04	+ 0·01	− 0·3	+ 0·4	9·1
85	90	71	,,	120911 123434	090665 093145	592·995	+ 216·16	1078·82	+ 214·87	45·08	+ 3 34·8	+ 0·07	+ 0·04	+ 0·6	− 4·1	10·4
,,	,,	,,	19	123171 120686	093302 090830	593·044	+ 216·39	1078·92	+ 215·10	45·09	+ 3 35·1	—	—	—	—	—
86	91	72	7	121158 123652	093967 096469	596·860	+ 160·90	1085·41	+ 159·59	45·50	+ 2 39·5	+ 0·01	− 0·01	− 0·3	− 0·9	9·4
87	92	74	,,	121620 124137	091033 093510	604·759	+ 210·06	1100·18	+ 208·72	46·51	+ 3 28·7	+ 0·11	+ 0·12	+ 1·6	− 1·2	10·5
88	93	75	,,	121865 124386	085950 088427	608·875	+ 294·81	1108·20	+ 293·45	47·10	+ 4 53·4	+ 0·11	+ 0·09	+ 1·2	+ 0·1	10·0
89	94	—	,,	122917 125450	123360 125869	626·505	− 329·70	1135·22	− 331·13	48·56	− 5 31·4	—	− 0·09	—	− 3·5	11·3
90	96	79	,,	123156 125668	101727 107229	+ 631·154	− 18·63	+ 146·126	− 20·09	+1 49·46	− 20·2	+ 0·14	+ 0·11	− 0·3	− 0·6	9·3

TABLE VI.—CATALOGUE OF 223 STARS IN THE CLUSTER χ PERSEI—*continued*.

No.	No. in Pihl.	No. in Vogel.	Sheet.	*H.*	*V.*	*X.*	*Y.*	Apparent Δα from Centre.	Apparent Δδ from Centre.	Mean Δα from No. 5.	Mean Δδ from No. 5.	Δα.		Δδ.		Mag.
												D − *V.*	*D* − *P.*	*D* − *V.*	*D* − *P.*	
										m. *s.*	′ ″	*s.*	*s.*	″	″	
91	95	78	7	123063 125556	080335 082821	+ 628″·605	+ 388″·41	+ 1145″·06	+ 386″·97	+1 49·60	+ 6 27″·0	+ 0·31	+ 0·22	− 0″·9	+ 1″·0	11·4
92	98	80	,,	123529 126049	100100 102588	636·603	+ 58·70	1156·82	+ 57·21	50·21	+ 57·1	+ 0·04	+ 0·01	− 0·4	+ 0·6	8·5
93	97	—	,,	123722 126244	126658 129123	639·853	− 384·38	1158·95	− 385·88	50·11	− 6 26·2	—	+ 0·37	—	− 5·8	11·6
,,	,,	—	19	126062 123574	129271 126868	641·203	− 384·47	1161·40	− 385·97	50·27	− 6 26·2	—	—	—	—	—
94	99	—	7	123374 125943	036714 039247	634·421	+ 1114·83	1161·88	+ 1113·32	51·11	+18 33·6	—	− 0·14	—	0·0	10·8
95	100	82	,,	124066 126563	097325 099819	645·352	+ 104·95	1173·11	+ 103·42	51·32	+ 1 43·3	+ 0·06	− 0·04	− 1·0	− 0·5	8·8
96	101	—	8	127198 124675	112281 109792	655·567	− 102·72	1189·88	− 104·30	52·32	− 1 44·5	—	+ 0·12	—	− 1·2	11·6
97	102	86	,,	127164 124639	101373 098879	654·983	+ 79·38	1190·39	+ 77·80	52·44	+ 1 17·7	0·00	− 0·13	− 0·6	− 0·4	9·0
98	103	—	8	127611 125089	115736 113263	662·466	− 160·53	1201·87	− 162·14	53·08	− 2 42·3	—	− 0·02	—	− 0·5	11·5
99	104	89	,,	128045 125490	081775 079301	669·414	+ 406·06	1219·54	+ 404·40	54·57	+ 6 44·4	− 0·25	− 0·23	− 0·6	+ 0·3	11·4
100	105	91	,,	128346 125820	057045 054562	674·680	+ 818·19	1232·88	+ 816·51	55·68	+13 36·7	− 0·07	0·00	− 0·5	+ 1·0	10·3
101	106	—	,,	129591 127050	121554 119058	695·293	− 257·44	1260·52	− 259·22	56·94	− 4 19·4	—	+ 0·13	—	+ 0·6	10·6
102	107	95	,,	129575 127039	091851 089383	695·076	+ 238·00	1264·73	+ 236·22	57·49	+ 3 56·2	− 0·01	− 0·12	− 0·1	− 0·4	11·0
103	109	—	,,	130337 127800	165860 163363	707·757	− 996·67	1276·20	− 998·48	57·59	− 16 38·9	—	− 0·24	—	− 2·6	11·2
104	108	96	,,	129571 127044	078141 075667	695·076	+ 466·63	1266·86	+ 464·84	57·76	+ 7 44·9	− 0·03	− 0·05	− 0·7	+ 0·4	10·6
105	110	98	,,	130048 127521	067892 065418	703·025	+ 637·42	1282·97	+ 635·59	1 58·92	+ 10 35·7	+ 0·03	+ 0·04	+ 0·7	+ 0·5	8·5
106	112	100	,,	131068 128515	099952 097458	719·805	+ 103·10	1308·43	+ 101·20	2 0·33	+ 1 41·1	+ 0·11	+ 0·09	− 0·1	+ 0·2	8·8
107	113	101	,,	131137 128589	084042 081571	721·005	+ 368·25	1313·17	+ 366·33	0·80	+ 6 6·3	− 0·15	− 0·18	− 0·2	+ 1·0	10·5
108	114	102	,,	131459 128923	084843 082342	726·471	+ 355·14	1322·99	+ 353·19	1·44	+ 5 53·2	− 0·07	− 0·13	+ 0·3	+ 1·4	10·4
109	116	—	9	129584 132097	151124 153563	739·368	− 790·28	1335·22	− 792·27	1·65	− 13 12·7	—	− 0·24	—	− 2·5	11·5
110	115	104	8	131706 129203	097183 094697	730·853	+ 149·22	1328·96	+ 147·26	1·73	+ 2 27·2	+ 0·07	+ 0·07	− 0·7	− 0·2	11·2

116	122	119	,,	133933 136455	079246 081741	811·922	+ 408·65	1479·19	+ 406·22	11·91	+ 6 46·2	+ 0·07	+ 0·01	+ 0·1	− 0·2	9·0
117	123	121	,,	134235 136753	073004 075506	816·921	+ 512·62	1489·44	+ 510·16	12·64	+ 8 30·2	+ 0·01	+ 0·01	− 0·2	− 0·9	10·2
118	124	122	,,	135626 138179	092855 095366	840·384	+ 181·58	1528·49	+ 178·98	15·07	+ 2 58·9	+ 0·14	+ 0·06	− 0·4	− 0·9	10·5
119	125	—	,,	136346 138851	115144 117603	851·982	− 189·95	1545·35	− 192·61	15·99	− 3 12·8	—	+ 0·06	—	− 2·2	9·9
120	126	—	,,	137698 140215	111900 120384	874·594	− 236·16	1585·82	− 238·96	18·66	− 3 59·2	—	− 0·02	—	− 0·5	9·8
121	127	125	,,	137658 140181	106442 108914	873·978	− 44·82	1586·95	− 47·63	18·85	− 47·8	+ 0·03	+ 0·03	− 0·4	− 0·7	9·4
122	128	—	,,	138339 140882	142604 145106	885·492	− 648·70	1600·75	− 651·56	19·44	− 10 51·9	—	− 0·22	—	− 1·5	11·1
123	130	—	10	141418 138964	162833 160386	895·607	− 945·69	1615·52	− 948·60	20·25	− 15 49·1	—	− 0·35	—	− 3·9	11·6
124	129	127	9	138353 140860	085415 087901	885·426	+ 305·86	1611·89	+ 302·97	20·70	+ 5 2·9	− 0·13	− 0·19	+ 0·2	− 0·3	9·9
125	131	129	10	140936 138444	066555 064072	887·275	+ 660·69	1619·49	+ 657·78	21·38	+ 10 57·9	+ 0·12	+ 0·07	− 0·6	+ 0·2	10·8
126	132	130	,,	141387 138906	086784 084299	894·874	+ 323·54	1629·30	+ 320·59	21·85	+ 5 20·5	0·00	− 0·11	− 0·1	− 0·8	9·9
127	133	131	,,	141866 139389	065347 062855	902·889	+ 680·89	1648·23	+ 677·87	23·30	+ 11 18·0	− 0·05	0·00	− 1·8	+ 0·2	10·0
128	136	132	,,	143550 141066	103598 101127	930·901	+ 42·97	1691·38	+ 39·79	25·84	+ 39·7	− 0·03	+ 0·08	− 0·6	− 0·2	10·2
129	135	133	,,	143689 141220	115123 112620	933·334	− 149·14	1693·41	− 152·33	25·87	− 2 32·5	+ 0·04	+ 0·17	+ 0·7	− 0·7	10·1
130	137	134	,,	143668 141194	107735 105280	932·951	− 26·22	1694·24	− 29·42	26·00	− 29·6	+ 0·06	− 0·04	− 0·3	− 0·1	8·7
131	138	—	,,	144103 141611	040500 038011	940·049	+1094·85	1721·32	+1091·55	28·40	+ 18 11·8	—	+ 0·11	—	− 0·1	9·6
132	139	135	,,	144832 142332	089076 086592	952·131	+ 285·33	1733·05	+ 281·99	28·75	+ 4 41·9	− 0·06	− 0·09	− 0·8	+ 0·1	10·4
133	140	—	,,	145887 143405	133237 130775	969·861	− 451·85	1755·79	− 455·29	29·87	− 7 35·6	—	+ 0·16	—	− 1·4	9·5
134	141	136	,,	145339 142876	083216 080747	960·879	+ 382·89	1750·22	+ 379·49	29·94	+ 6 19·5	− 0·03	− 0·03	+ 0·4	− 0·2	9·9
135	142	137	,,	146001 143515	093209 090726	971·727	+ 216·39	1767·82	+ 212·91	31·03	+ 3 32·8	− 0·02	+ 0·06	− 0·6	− 0·7	9·5
136	143	138	,,	146612 144114	094622 092130	981·809	+ 192·89	1785·85	+ 189·34	32·22	+ 3 9·2	− 0·13	− 0·19	− 1·6	+ 0·1	10·9
137	144	139	,,	147798 145294	073477 070971	1001·522	+ 545·55	1826·45	+ 541·84	35·12	+ 9 1·9	+ 0·16	+ 0·10	− 0·2	+ 0·2	7·7
138	145	140	11	145619 148171	081630 084115	1006·504	+ 368·62	1833·14	+ 364·89	35·48	+ 6 4·9	− 0·04	− 0·11	− 1·7	− 2·6	11·1
139	146	—	,,	147475 149995	169828 172297	1037·165	−1102·82	1868·70	−1106·71	37·05	− 18 27·2	—	− 0·14	—	− 3·1	10·0
140	147	142	,,	147113 149655	081685 084165	+1031·317	+ 367·75	+ 1878·30	+ 363·82	+ 2 38·50	+ 6 3·8	− 0·02	− 0·03	− 1·9	− 0·4	10·0

TABLE VI.—CATALOGUE OF 223 STARS IN THE CLUSTER χ PERSEI—*continued.*

No.	No. in Pihl.	No. in Vogel.	Sheet.	*H.*	*V.*	*X.*	*Y.*	Apparent Δα from Centre.	Apparent Δδ from Centre.	Mean Δα from No. 5.	Mean Δδ from No. 5.	Δα. *D* − *V*	Δα. *D* − *P.*	Δδ. *D* − *V.*	Δδ. *D* − *P.*	Mag.
										m. *s.*		*s.*	*s.*			
141	148	143	11	147050 149625	065936 068420	+1030″·550	+ 630″·20	+1880″·55	+ 626″·27	+2 38·78	+ 10 26″·4	+ 0·09	+ 0·06	− 1″·7	+ 0″·2	10·7
142	149	—	,,	147985 150523	121002 123443	1045·814	− 287·95	1895·56	− 291·96	39·29	− 4 52·2	—	+ 0·09	—	− 0·5	9·3
143	150	—	,,	147967 150497	040288 042754	1045·447	+1057·65	1913·78	+1053·58	41·23	+ 17 33·8	—	+ 0·11	—	+ 1·4	7·2
144	151	145	,,	148676 151208	079884 082398	1057·262	+ 397·49	1925·99	+ 393·36	41·69	+ 6 33·4	+ 0·05	− 0·08	+ 1·0	− 0·2	10·7
145	152	146	,,	148627 151186	059997 062468	1056·679	+ 729·23	1929·64	+ 725·10	42·11	+ 12 5·2	− 0·02	+ 0·05	− 2·3	− 0·1	9·9
146	153	—	,,	149367 151906	094272 096762	1068·843	+ 157·74	1943·64	+ 153·53	42·73	+ 2 33·5	—	+ 0·10	—	− 0·2	10·0
147	154	150	,,	149526 152083	097660 100184	1071·643	+ 100·95	1947·91	+ 96·73	42·99	+ 1 36·7	+ 0·06	+ 0·07	− 0·8	+ 0·8	10·8
148	155	154	,,	151048 153594	096322 098801	1096·888	+ 123·65	1994·14	+ 119·22	46·09	+ 1 59·1	− 0·07	− 0·03	− 1·6	− 1·3	9·8
149	156	155	,,	151704 154220	098682 101193	1107·570	+ 84·00	2012·97	+ 79·49	47·31	+ 1 19·4	+ 0·01	− 0·25	− 1·0	+ 1·3	10·9
150	158	—	,,	154097 156632	140227 142697	1147·529	− 609·12	2075·01	− 613·93	51·08	− 10 14·3	—	− 0·08	—	− 0·3	10·7
151	159	162	11	154074 156627	080716 083213	1147·296	+ 383·75	2089·78	+ 378·89	52·60	+ 6 18·9	+ 0·03	− 0·01	− 0·1	+ 0·7	8·0
152	160	163	12	157013 154300	089091 086612	1153·945	+ 285·95	2100·38	+ 281·05	53·26	+ 4 41·0	+ 0·02	+ 0·02	0·0	+ 0·1	8·9
153	162	—	,,	159736 157063	154494 152028	1199·586	− 805·60	2166·03	− 810·85	57·05	− 13 31·3	—	− 0·25	—	− 1·8	10·6
154	163	165	,,	159491 156814	096941 094444	1195·488	+ 155·16	2173·89	+ 149·90	2 58·09	+ 2 29·8	− 0·08	− 0·02	− 1·9	0·0	10·4
155	164	166	,,	160230 157552	079558 077053	1207·768	+ 445·09	2200·91	+ 439·70	3 0·06	+ 7 19·7	0·00	− 0·01	− 0·1	0·0	10·4
156	165	167	,,	160160 157448	069786 067300	1206·319	+ 607·80	2200·91	+ 602·42	0·14	+ 10 2·5	− 0·08	− 0·16	− 1·0	+ 1·0	10·4
157	166	—	,,	161380 158697	112070 109580	1226·865	− 97·38	2226·79	− 102·90	1·49	− 1 43·1	—	[+ 1·25]	—	[+ 2·3]	11·2
158	167	—	,,	160967 158260	039203 036703	1219·800	+1117·49	2233·91	+1111·95	2·61	+ 18 32·2	—	− 0·29	—	+ 2·4	11·0
159	168	170	,,	162921 160186	080393 077907	1252·077	+ 431·01	2281·41	+ 425·22	5·41	+ 7 5·2	+ 0·04	+ 0·01	+ 0·3	+ 0·6	8·5
160	170	—	,,	163404 160668	113112 110645	1260·109	− 114·94	2286·85	− 120·77	5·48	− 2 1·0	—	− 0·21	—	− 3·7	11·1
161	169	171	,,	162715 159983	057550 055085	1248·678	+ 811·45	2281·62	+ 805·67	5·63	+ 13 25·8	− 0·11	− 0·03	0·5	+ 0·1	10·1

				161156	039529											
166	175	—	13	162455	114285	1289·404	− 176·14	2338·95	− 182·24	8·94	− 3 2·5	—	− 0·16	—	− 0·5	10·9
				164983	116806											
167	177	—	,,	164085	113888	1316·633	− 169·69	2388·44	− 176·05	12·24	− 2 56·3	—	− 0·39	—	− 0·5	11·2
				166625	116429											
168	176	—	,,	163120	036821	1300·619	+1115·42	2381·88	+1109·12	12·50	+ 18 29·3	—	− 0·16	—	+ 1·5	9·8
				165666	039336											
169	178	—	,,	165263	107181	1336·313	− 57·60	2426·14	− 64·16	14·82	− 1 4·3	—	+ 0·02	—	− 0·2	9·9
				167813	109706											
170	179	172	,,	165192	076520	1335·112	+ 453·93	2433·11	+ 447·35	15·56	+ 7 27·3	− 0·02	− 0·03	+ 0·1	+ 1·6	9·2
				167740	079031											
171	180	—	,,	164589	027091	1325·265	+1277·79	2429·92	+1271·23	15·80	+ 21 11·6	—	− 0·16	—	− 1·2	10·7
				167160	029590											
172	181	174	,,	166617	055396	1358·575	+ 806·13	2482·31	+ 799·29	19·03	+ 13 19·4	− 0·24	+ 0·03	− 2·5	+ 0·3	10·5
				169135	057881											
173	183	—	,,	168352	149267	1387·287	− 760·29	2505·74	− 767·30	19·75	− 12 47·7	—	+ 0·05	—	− 1·4	8·9
				170851	151820											
174	185	175	,,	167823	068898	1378·538	+ 580·48	2514·60	+ 573·46	21·07	+ 9 33·5	− 0·19	+ 0·08	0·0	+ 1·0	10·6
				170328	071468											
175	186	—	,,	168794	037777	1395·218	+1099·09	2554·80	+1091·85	24·02	+ 18 12·0	—	− 0·31	—	− 2·1	10·7
				171361	040338											
176	187	—	,,	170084	083259	1416·265	+ 341·28	2578·86	+ 333·88	25·22	+ 5 33·8	—	− 0·15	—	+ 0·4	10·7
				172600	085807											
177	188	—	,,	171395	137931	1437·978	− 570·77	2600·90	− 578·32	26·20	− 9 38·7	—	− 0·42	—	− 3·1	10·2
				173899	140440											
178	189	—	,,	174197	169948	1484·502	− 1104·94	2674·57	−1112·94	30·82	− 18 33·5	—	− 0·20	—	− 2·2	10·6
				176683	172474											
179	190	—	,,	173874	070930	1479·520	+ 546·87	2698·09	+ 538·78	33·28	+ 8 58·8	—	− 0·39	—	+ 1·1	11·0
				176408	073471											
180	191	—	14	176506	047618	1481·186	+ 977·29	2709 75	+ 969·14	34·29	+ 16 9·3	—	− 0·37	—	+ 2·4	10·7
				173979	045133											
181	192	—	,,	177292	053064	1494·051	+ 886·56	2731·44	+ 878·27	35·69	+ 14 38·4	—	− 0·10	—	− 2·0	10·9
				174738	050575											
182	193	—	,,	177996	044715	1505·849	+ 1025·83	2755·89	+1017·40	37·39	+ 16 57·6	—	− 0·23	—	− 0·9	10·7
				175449	042211											
183	195	—	,,	178790	079152	1518·947	+ 451·72	2768·05	+ 443·20	37·89	+ 7 23·2	—	− 0·29	—	− 1·4	10·8
				176226	076687											
184	194	—	,,	178804	074551	1519·330	+ 528·45	2770·32	+ 519·92	38·09	+ 8 39·9	—	− 0·08	—	+ 1·2	9·5
				176257	072081											
185	196	—	,,	178940	081224	1521·829	+ 417·17	2772·59	+ 408·52	38·18	+ 6 48·5	—	− 0·19	—	+ 0·5	10·0
				176420	078760											
186	197	—	,,	180479	063110	1547·408	+ 719·12	2825·48	+ 710·25	41·87	+ 11 50·3	—	+ 0·02	—	+ 1·0	9·2
				177946	060634											
187	198	—	,,	180470	049312	1547·275	+ 949·13	2830·05	+ 940·23	42·29	+ 15 40·4	—	− 0·05	—	+ 1·6	8·8
				177939	046817											
188	199	—	,,	183330	163847	1594·700	− 961·64	2876·09	− 970·89	44·33	− 16 11·4	—	− 0·41	—	− 1·3	9·8
				180766	161417											
189	—	—	,,	183460	124111	1596·782	− 297·70	2893·87	− 307·04	45·88	− 5 7·3	—	—	—	—	[11·5]
				180886	121567											
190	201	—	,,	183604	114310	+ 1599·515	− 134·98	+ 2902·29	− 144·37	+ 3 46·53	− 2 24·6	—	+ 0·19	—	− 0·6	10·6
				181070	111870											

TABLE VI.—CATALOGUE OF 223 STARS IN THE CLUSTER χ PERSEI—*continued.*

No.	No. in Pihl.	No. in Vogel.	Sheet.	*H.*	*V.*	*X.*	*Y.*	Apparent Δα from Centre.	Apparent Δδ from Centre.	Mean Δα from No. 5.	Mean Δδ from No. 5.	Δα.		Δδ.		Mag.
												D − *V.*	*D* − *P.*	*D* − *V.*	*D* − *P.*	
										m. *s.*	′ ″		*s.*			
191	202	—	14	184444	055411	+1613″·763	+ 847″·25	+ 2949″·42	+ 837″·59	+3 50·20	+ 13 57″·7	—	− 0·15	—	+ 0″·4	10·6
				181937	052949											
192	204	—	,,	185759	097434	1635·659	+ 147·17	2974·01	+ 137·32	51·47	+ 2 17·2	—	− 0·19	—	− 0·4	10·0
				183246	094932											
193	205	—	19	188097	155709	1674·535	− 824·51	3023·19	− 834·72	54·19	− 13 55·2	—	− 0·32	—	− 2·7	10·4
				185599	153173											
194	207	—	15	173638	160757	1682·101	− 955·67	3038·83	− 965·95	54·84	− 16 6·5	—	− 0·18	—	− 0·1	10·4
				176185	163207											
195	206	—	18	187551	106059	1666·253	+ 2·52	3026·42	− 7·69	54·88	− 7·9	—	− 0·01	—	− 0·5	9·3
				185054	103515											
,,	,,	—	14	187590	106106	1666·054	+ 2·62	3026·06	− 7·58	[54·86]	[− 7·8]	—	—	—	—	—
				185059	103587											
,,	,,	—	15	172764	103311	1666·604	+ 2·55	3027·06	− 7·66	[54·91]	[− 7·9]	—	—	—	—	—
				175200	105808											
196	208	—	,,	172500	070123	1662·605	+ 556·42	3032·14	+ 546·20	55·55	+ 9 6·1	—	− 0·28	—	− 1·1	10·6
				174984	072567											
197	209	—	,,	174124	071923	1689·766	+ 525·65	3080·97	+ 515·09	58·79	+ 8 35·0	—	− 0·10	—	− 0·7	10·6
				176619	074461											
198	210	—	,,	174219	075031	1691·266	+ 474·08	3082·54	+ 463·52	3 58·86	+ 7 43·4	—	− 0·52	—	+ 1·1	11·4
				176704	077540											
199	211	—	,,	175731	127486	1716·528	− 400·37	3108·51	− 411·15	4 0·13	− 6 51·5	—	+ 0·07	—	+ 1·5	10·9
				178223	129908											
200	212	—	,,	176349	144110	1726·660	− 678·44	3120·51	− 689·31	0·77	− 11 29·8	—	− 0·17	—	− 1·0	9·9
				178818	146603											
201	214	—	,,	177680	150530	1748·805	− 784·95	3158·06	− 796·10	3·22	− 13 16·6	—	− 0·17	—	− 0·8	10·5
				180144	152953											
202	215	—	,,	178584	053977	1764·020	+ 824·88	3223·46	+ 813·34	8·45	+ 13 33·4	—	− 0·02	—	+ 0·3	10·4
				181063	056485											
203	216	—	,,	180610	156802	1798·514	− 889·65	3245·33	− 901·42	8·98	− 15 2·0	—	− 0·54	—	− 1·8	10·4
				183173	159241											
204	217	—	,,	181492	075435	1812·678	+ 467·84	3303·63	+ 455·70	13·61	+ 7 35·6	—	+ 0·17	—	− 0·6	10·8
				183988	077884											
205	218	—	,,	182418	119874	1827·925	− 273·42	3313·30	− 285·66	13·85	− 4 46·0	—	+ 0·12	—	− 0·5	9·9
				184890	122312											
206	219	—	16	185200	119607	1832·841	− 228·41	3323·31	− 240·73	14·55	− 4 1·0	—	− 0·07	—	+ 2·0	11·1
				182753	117196											
207	220	—	,,	185648	134622	1840·323	− 478·64	3330·75	− 491·03	14·91	− 8 11·4	—	− 0·02	—	− 0·1	9·7
				183202	132160											
208	221	—	,,	186398	150103	1852·787	− 736·36	3346·99	− 748·88	15·85	− 12 29·4	—	− 0·29	—	− 0·2	10·0
				183946	147564											
209	222	—	17	184940	042451	1848·055	+1018·87	3381·84	+1006·18	19·13	+ 16 46·3					
				182531	039977											

213	225	—	,,	188479 185998	048561 046137	1887·247	+ 956·27	3451·95	+ 943·04	23·77	+ 18 18·1	—	[illegible]		+ 2·1	10·7
214	227	—	,,	189670 187262	096133 093720	1907·728	+ 163·35	3469·03	+ 149·95	24·48	+ 2 29·8	—	+ 0·06	—	+ 2·1	10·7
215	228	—	15	187777 190237	114787 117226	1917·225	− 188·54	3477·30	− 202·02	24·83	− 3 22·4	—	− 0·04	—	− 0·5	10·4
216	229	—	16	190471 188067	126416 123962	1921·125	− 341·71	3480·45	− 355·22	24·96	− 5 55·6	—	− 0·01	—	+ 0·1	9·3
217	230	—	,,	189709 187279	053690 051291	1908·194	+ 870·65	3488·04	+ 857·13	26·12	+ 14 17·2	—	− 0·04	—	+ 1·5	9·1
218	231	—	17	189966 187536	040790 038315	1931·723	+ 1046·58	3535·65	+ 1032·70	29·40	+ 17 12·8	—	− 0·17	—	+ 2·7	9·1
219	232	—	16	194490 192053	114176 111787	1987·896	− 137·94	3606·79	− 152·44	33·50	− 2 32·7	—	+ 0·11	—	+ 1·1	8·7
220	233	—	,,	195276 192843	089117 086671	2001·043	+ 280·66	3641·84	+ 265·91	36·06	+ 4 25·8	—	+ 0·07	—	+ 1·2	9·5
221	234	—	,,	199657 197207	154941 152529	2073·998	− 818·07	3744·27	− 833·74	42·33	− 13 54·3	—	− 0·06	—	0·0	9·8
222	235	—	,,	200187 197778	135451 133028	2083·179	− 492·80	3769·81	− 508·67	44·18	− 8 29·1	—	+ 0·05	—	0·0	10·2
223	236	—	17	197942 195378	049966 047426	+ 2063·666	+ 894·15	+ 3772·84	+ 878·33	+ 4 45·13	+ 14 38·4	—	− 0·01	—	+ 1·7	10·0

Table VII.—Annual Precession in $\Delta\alpha$ and $\Delta\delta$.

$\Delta\alpha$.	$\Delta''_{\alpha\alpha}$		$\Delta\delta$.	$\Delta''_{\alpha\delta}$.		$\Delta\alpha$.	$\Delta''_{\delta\alpha}$.	
m. s.	s.		′	s.		m. s.	″	
+0 0	+0·0000		+ 0	+0·0000		+0 0	−0·000	
10	·0012		1	·0007		10	·008	
20	·0025		2	·0014		20	·016	
30	·0037		3	·0021		30	·024	
40	·0049		4	·0028		40	·032	
50	·0062		5	·0035		50	·040	
1 0	·0074		6	·0042		1 0	·048	
10	·0086	*P. P.*	7	·0049	*P. P.*	10	·056	*P. P.*
20	·0099	s.	8	·0056		20	·064	s.
30	·0111	10 12·34	9	·0063	1′·0 7″·01	30	·072	10 7″·97
40	·0123		10	·0070		40	·080	
50	·0136	9 11			0·9 6	50	·088	9 7
		8 10	11	·0077	·8 6			8 6
2 0	·0148	7 9	12	·0084	·7 5	2 0	·096	7 6
10	·0160	6 7	13	·0091	·6 4	10	·104	6 5
20	·0173	5 6	14	·0098	·5 4	20	·112	5 4
30	·0185	4 5	15	·0105	·4 3	30	·120	4 3
40	·0197	3 4			·3 2	40	·128	3 2
50	·0210	2 2	16	·0112	·2 1	50	·135	2 2
		1 1	17	·0119	·1 1			1 1
3 0	·0222		18	·0126		3 0	·143	
10	·0234		19	·0133		10	·151	
20	·0247		20	·0140		20	·159	
30	·0259					30	·167	
40	·0271		21	·0147		40	·175	
50	·0284		22	·0154		50	·183	
			23	·0161				
4 0	·0296		24	·0168		4 0	·191	
10	·0308		+ 25	+ ·0175		10	·199	
20	·0321					20	·207	
30	·0333					30	·215	
40	·0346					40	·223	
50	·0358					50	·231	
+5 0	+0·0370					+5 0	·239	

The annual precession in R. A. is equal to $\Delta''_{\alpha\alpha} + \Delta''_{\alpha\delta}$.

" " " " δ " $\Delta''_{\delta\alpha}$.

For negative values of $\Delta\alpha$ and $\Delta\delta$ the quantities $\Delta''_{\alpha\alpha}$, $\Delta''_{\alpha\delta}$, and $\Delta''_{\delta\alpha}$ are to be taken with an opposite sign to that given in the Tables.

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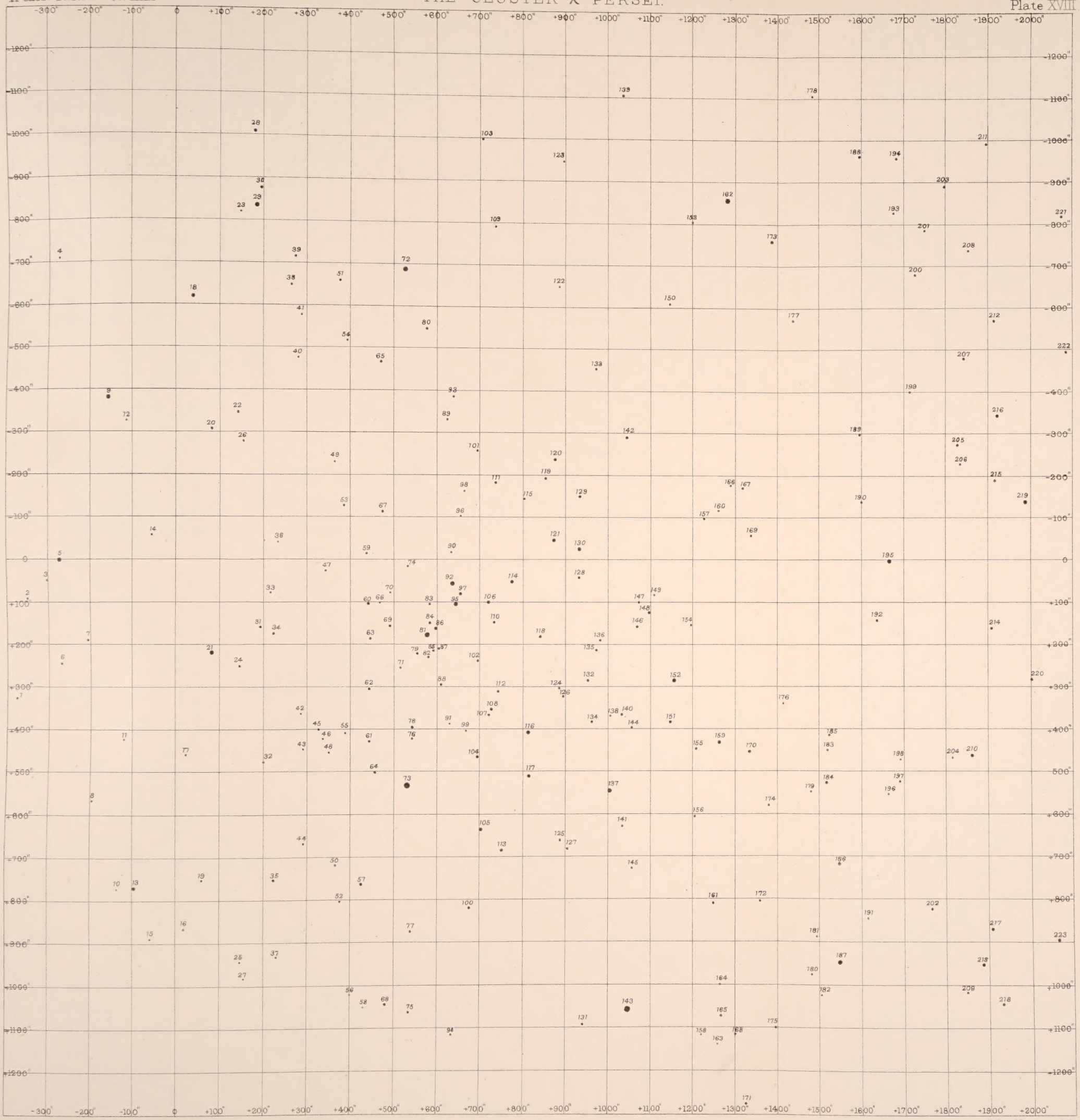

In this map the positions of the Stars are plotted down with the values of X & Y from Table VI.

West, Newman. lith